The Fair Valuation of Life Insurance Participating Policies: The Mortality Risk Role

Massimiliano Politano

Department of Mathematics and Statistics
University of Naples "Federico II"
Via Cinthia, Monte S.Angelo - Napoli - Italy
(e-mail: politano@unina.it)

Abstract. This paper analyses the role of the term structure of interest and mortality rates for life insurance participating policies. In particular, aim of this work is to determine the fair valuation of such a policy by modelling mortality risk by means of a Lee Carter type methodology. Numerical results are investigated in order to determine the fair value accounting impact on reserve evaluations.
Keywords: Participating policies, Fair pricing, Lee Carter Methodology, CIR-Black and Scholes framework.

1 Introduction

In life insurance, actuaries have traditionally valued premiums and reserves using deterministic mortality intensity, which is a function of the age of the insured only, and some hypothesis on the dynamics of interest rates. However, since neither the interest rates nor the mortality intensity is deterministic, life insurance companies are essentially exposed to three kinds of risk: the financial risk, the systematic mortality risk, referring to the future development of the underlying mortality intensity, and unsystematic mortality risk, referring to a possible adverse development of the policyholders mortality. It must be pointed out that only the third kind of risk can be controlled by means of portfolio diversification. Since insurance contracts often run for a very long time, a mortality intensity which seems to be prudential at the time of issue, might turn out not to be so. An analogous phenomenon has been observed for the interest rates in the last two decades where we have experienced large drops in the stock prices and low returns on bonds. However, the systematic mortality risk is of different character than the financial risk. While the assets on the financial markets are very volatile, changes in the mortality intensity seems to occur more slowly. Thus, the financial market poses an immediate problem, whereas the level of mortality intensity poses a more long term, but also more permanent problem. This difference could be the reason why emphasis so far has been on the financial markets. In recent years, some of this attention has shifted towards valuation models that fully capture the interest and mortality rates dynamics. In this context, the contribute of the International Accounting standard Board was very important. It defines the Fair Value as "An estimate of an exit price determined by market interactions". At this proposal, it must be remembered that IASB allows for using stochastic models in order to estimate future cash flows. In practice, the problem of determine the market value of insurance liabilities is posed. In this field, it must be remembered the papers of Grosen-Jorgensen [Grosen and Jorgensen, 2000], Bacinello [Bacinello, 2001], Milevsky-Promislow [Milevsky and Promislow, 2001], Ballotta-Haberman [Ballotta and Haberman, 2003]. Here we analyse, in a Lee Carter mortality context, one of the most common life fe insurance policies present on the Italian insurance market, the so called revaluable policy. This policy, of endowment type, has the peculiarity that the insurance company, at the end of each year, grants a bonus which is credited to the mathematical reserve and depends on the performance of an investment portfolio. This bonus is determined in such a way that the total interest credited to the insured is equal to a give percentage of the annual return of the reference portfolio and anyway does not fall below the minimum interest rate guaranteed. Thus, the revaluable policy is of participating type. The paper is organised as follows: section 2 develops the framework for the valuation of the policy. in section 3, the Lee Carter model for the mortality risk is introduced, in section 4 the financial market model is presented. A numerical evidence is offered in section 5.

2 The Model

Let us consider an endowment policy issued at time 0 and maturing at time ξ, with initial sum insured C_0. Moreover, let us define $\{r_t; t = 1, ..., \xi\}$ and $\{\mu_{x+t}; t = 1, ..., \xi\}$ the random spot rate process and the mortality process respectively, both of them measurable with respect to the filtrations $\mathcal{F}^r$ and $\mathcal{F}^\mu$. The above mentioned processes are defined on a unique probability space $(\Omega, \mathcal{F}^{r,\mu}, P)$ such that $\mathcal{F}^{r,\mu} = \mathcal{F}^r \cup \mathcal{F}^\mu$. For the revaluable endowment policy, we assume that, in case of single premium, at the end of the t-th year, if the contract is still in force the mathematical reserve is adjusted at a rate ρ_t defined as follows [Bacinello, 2001]

$$\rho_t = max\left\{\frac{\eta S_t - i}{1+i}, 0\right\} \qquad t = 1, ...\xi \tag{1}$$

The parameter η, $0 \leq \eta \leq 1$, denotes the constant partecipating level, and S_t indicates the annual return of the reference portfolio. The relation (1) explains the fact that the total interest rate credited to the mathematical reserve during the t-th year, is the maximum between ηS_t and i, where i is the minimum rate guaranteed to the policyholder. Since we are dealing with a single premium contract, the bonus credited to the mathematical reserve implies a proportional adjustment at the rate ρ_t, also of the sum insured.

Denoting by C_t, $t = 1, ..., \xi$, the benefit paid at time t if the insured dies between ages x+t-1, x+t or, in case of survival, for $t = \xi$, the following recursive relation holds for the benefits of successive years

$$C_t = C_{t-1}\left(1 + \rho_t\right) \qquad t = 1, ..., \xi$$

The iterative expression for them is instead

$$C_t = C_0 \prod_{j=1}^{t} \left(1 + \rho_j\right) \qquad t = 1, ..., \xi$$

where we have indicated by ϕ_t the readjustment factor

$$\phi_t = \prod_{j=1}^{t} \left(1 + \rho_j\right) \qquad t = 1, ..., \xi$$

In this context, as the elimination of the policyholder can happen in case of death in the year $t \in [0, \xi\, [$ or in case of survival $t = \xi$ the liability borne out by the insurance company can be expressed in this manner

$$W_0^L = \sum_{t=0}^{\xi} C_t \;{}_{t-1/1}Y_x + C_\xi \;{}_\xi J_x \tag{2}$$

where

$${}_{t-1/1}Y_x = \begin{cases} e^{-\Delta(t)} \; if \; t-1 < T_x \leq t \\ 0 \qquad \text{otherwise} \end{cases} \qquad {}_\xi J_x = \begin{cases} 0 \qquad if \; 0 < T_x < \xi \\ e^{-\Delta(\xi)} \qquad T_x \geq \xi \end{cases}$$

In the previous expression T_x is a random variable which represents the remaining lifetime of a insured aged x, $\Delta(t) = \int_0^t r_u du$ is the accumulation function of the spot rate.

3 Mortality Risk modeling

for the dynamics of the process $\{\mu_{x+t}; t = 1, 2, ...\}$, we propose to choose a model based on the Lee Carter methodology. According to the traditional actuarial approach, the survival function of the random variable T_x is given by [Milevsky and Promislow, 2001]

$${}_\xi p_x = P\left(T_x > \xi / \mathcal{F}_0^\mu\right)$$

where $\mathcal{F}_0^\mu$ represents the mortality informative structure available at time 0. If we make the hypothesis of the time dependence of the mortality intensity and we define $\mu_{x+t:t}$ the mortality intensity for an individual aged x+t, observed in the year t, it is possible to express the previous formula as follows [Ballotta and Haberman, 2003]

$$ {}_{\xi}p_x = E\left(exp\left\{-\int_0^{\xi}\mu_{x+t:t}dt\right\}/\mathcal{F}_0^{\mu}\right) \tag{3} $$

A widely used actuarial model for projecting mortality rates is the reduction factor model. This model has traditionally been formulated with respect to the conditional probability of dying in a year

$$ q\,(y,t) = q\,(y,0)\,RF\,(y,t) $$

where $q\,(y,0)$ represents the probability that a person aged y will die in the next year, based on the mortality experience for the base year 0 and correspondingly $q\,(y,t)$ relates to the future year t. Given the form of (3), it is considered a reduction factor approach for the mortality intensity so that

$$ \mu_{y:t} = \mu_{y:0}RF\,(y,t) \tag{4} $$

where $\mu_{y:0}$ is the mortality intensity of a person aged y in the base year 0, $\mu_{y:t}$ is the mortality intensity for a person attaining age y in the future year t, and the reduction factor is the ratio of the mortality intensity. It is possible to target RF, in a Lee Carter approach, $\mu_{y:0}$ being completely specified. Thus, $\mu_{y:0}$ is estimated

$$ \widehat{\mu}_{y:0} = \sum_t d_{y:t}/\sum_t e_{y:t} $$

where $d_{y:t}$ denotes the number of deaths at age y and time t and $e_{y:t}$ indicates the matching person years of exposure to the risk of death. Taking the logarithm of equation (4) and defining

$$ \alpha_y = \log\left(\mu_{y:0}\right) \qquad \log\left\{RF\,(y,t)\right\} = \beta_y k_t $$

the Lee Carter structure is reproduced [Renshaw and Haberman, 2003]. In fact the Lee Carter model for death rates is given by

$$ \ln\left(m_{yt}\right) = \alpha_y + \beta_y k_t + \epsilon_{yt} \tag{5} $$

where m_{yt} denotes the central mortality rate for age y at time t, α_y describes the shape of the age profile averaged over time, k_t is an index of the general level of mortality while β_y describes the tendency of mortality at age y to change when the general level of mortality k_t changes. ϵ_{yt} denotes the error. For this model, the strategy is to estimate the values for α_y, β_y, k_t on the historical data for the population in question, the difficulty concerns the fact that the quantities on the right hand of (5) are not directly observable. therefore, denoting by n the number of observable periods and $t = t_1, t_2, ..., t_n$, the parameters are normalized requiring that [Lee, 2000]

$$\sum_t k_t = 0 \qquad \sum_y \beta_y = 1$$

so that

$$\widehat{\alpha}_y = \ln\left[\left(\prod_t \widehat{\mu}_{yt}\right)^{\frac{1}{n}}\right] \qquad \widehat{k}_t = \sum_{y=0}^{\omega} \left[\ln\left(\widehat{m}_{yt}\right) - \widehat{\alpha}_x\right]$$

The parameter β_y can be estimated by an ordinary regression between $\widehat{k}_t$ and $\ln\left(\widehat{m}_{yt}\right)$ In this framework, for our purposes, with $y = x + t$, one can use the following model for the time evolution of the hazard rate

$$\mu_{x+t:t} = \mu_{x+t:0}\ e^{\beta_{x+t}k_t}$$

4 Financial Risk modeling

For the process $\{r_t; t = 1, 2, ...\}$, we assume a mean reverting square root dynamics

$$dr_t = f^r\left(r_t, t\right) dt + l^r\left(r_t, t\right) dZ_t^r$$

where $f^r\left(r_t, t\right)$ is the drift of the process, $l^r\left(r_t, t\right)$ is the diffusion coefficient, Z_t^r is a standard Brownian Motion; in particular, in the CIR model the drift function and the diffusion coefficient are defined respectively as [Cox *et al.*, 1985]

$$f^r\left(r_t, t\right) = k\left(\theta - r_t\right) \qquad l^r\left(r_t, t\right) = \sigma_r\sqrt{r_t}$$

where k is the mean reverting coefficient, θ the long term period "normal" rate, σ_r the spot rate volatility. It must be pointed out that for pricing interest rate derivatives the Vasicek model is widely used. Nevertheless, this model assigns positive probability to negative values of the spot rate; for long maturities, this can have relevant effects and therefore the vasicek model appears inadequate to value life insurance policies. Clearly on the fair pricing of our policy, it is very important the specification of the reference portfolio dynamics. The diffusion process for this dynamics is given by the stochastic differential equation

$$dS_t = f^S\left(S_t, t\right) + g^S\left(S_t, t\right) dZ_t^S$$

where S_t denotes the price at time t of the reference portfolio Z_t^S is a Standard Brownian Motion with the property

$$Cov\left(dZ_t^r, dZ_t^S\right) = \varphi dt \qquad \varphi \in R$$

Since we assume a BS type model [Black and Scholes, 1973], we have

$$f^S(S_t,t) = \mu_S S_t \qquad g^S(S_t,t) = \sigma_S S_t$$

where μ_S is the continously compounded market rate, assumed to be deterministic and constant and σ_S is the constant volatility paremeter.

In this context, for the policy under consideration, the unit price in t with maturity ξ is given by

$$u(t,\xi) = E\left[(exp\left\{-\int_t^{\xi} r_u du\right\}\phi_t/\mathcal{F}_t^r\right]$$

where $\mathcal{F}_t^r$ represents the financial informative structure available on the market at time t.

5 Some Applications

The described model has been applied in order to analyse the temporal profile of the insurance liability. The next table compares the value of the mathematical reserve using, for an insured aged 40 with time to maturity 20 years, a technical basis given by a constant interest rate of 3% and the life table SIM92 (Statistics Italian Males 1992) with the values obtained in a fair value context using the mortality Italian data for the period 1947-1999 for evaluate the projection of the mortality factor. Moreover we use the 3 month T-bill January 1996 - January 2004 for determine the interest rate factor, the parameters $\mu_S = 0.03$ $\sigma_S = 0.20$ for the stochastic evolution of the reference fund. For the correlation coefficient φ, we have adopted a slightly negative value ($\varphi = -0.06$) coherently with the literature for the Italian stock market.

The table 1 puts in evidence that the introduction of a fair value accounting system determines a reduction in the level of the liability borne out by the fund specially in the first years. This is mainly caused by the historical trend of the bond market where we have experienced a continuous decrease of interest rates. About the influence of the demographic factor, we have performed a comparison of the reserve value using the fair value hypothesis for the financial factors, and the life tables SIM92, RG48 (projected General Accountancy 1948) and the one obtained with the LC methodology for the mortality rates.

The fourth column of table 2 puts in evidence that the life table SIM92 underestimates the reserve values and don't capture the improvements of the human life. The life table RG48 accomplishes the second function but slightly overestimates the reserve value with respect to LC forecasts (i.e fifth column of table 2). Finally, we have calculated the variation coefficient of the contract value (column six of table 2), depending on demographic component in order to offer a measure of riskiness in reference to the problem to calculate an adequate margin for mortality risk in a fair value context.

Year	WtL	FVWtL	Δ WtL	Δ WtL/WtL
0	0,00	0,00	0,00	0,00%
1	1168,66	987,29	181,37	15,52%
2	1199,34	1028,77	170,57	14,22%
3	1231,14	1072,07	159,07	12,92%
4	1264,11	1117,28	146,83	11,62%
5	1295,36	1164,48	130,88	10,10%
6	1333,78	1213,73	120,05	9,00%
7	1370,64	1265,10	105,54	7,70%
8	1408,94	1318,64	90,30	6,41%
9	1448,92	1374,38	74,54	5,14%
10	1490,69	1432,30	58,39	3,92%
11	1534,02	1492,36	41,66	2,72%
12	1579,38	1554,45	24,93	1,58%
13	1620,72	1618,32	2,40	0,15%
14	1683,43	1683,43	0,00	0,00%
15	1735,89	1749,78	-13,89	-0,80%
16	1791,20	1815,98	-24,78	-1,38%
17	1879,40	1880,91	-1,51	-0,08%
18	1911,16	1942,83	-31,67	-1,66%
19	1976,96	1992,16	-15,20	-0,77%
20	2047,04	2047,04	0,00	0,00%

Table 1. Reserves temporal profile

Year	SIM92	RG48	LC	LC vs. SIM92	LC vs. RG48	CV
0	0,00	0,00	0,00	0,00%	0,00%	0,00000
1	991,75	986,80	987,29	-0,45%	0,05%	0,00276
2	1031,16	1028,78	1028,77	-0,23%	0,00%	0,00134
3	1072,35	1072,63	1072,07	-0,03%	-0,05%	0,00026
4	1115,43	1118,44	1117,28	0,17%	-0,10%	0,00136
5	1160,48	1166,29	1164,48	0,34%	-0,16%	0,00255
6	1207,59	1216,23	1213,73	0,51%	-0,21%	0,00367
7	1256,84	1268,36	1265,10	0,66%	-0,26%	0,00470
8	1308,28	1322,69	1318,64	0,79%	-0,31%	0,00564
9	1362,09	1379,23	1374,38	0,90%	-0,35%	0,00644
10	1418,32	1437,94	1432,30	0,99%	-0,39%	0,00707
11	1476,97	1498,74	1492,36	1,04%	-0,43%	0,00751
12	1537,32	1561,47	1554,45	1,11%	-0,45%	0,00801
13	1600,02	1625,86	1618,32	1,14%	-0,46%	0,00823
14	1664,60	1691,48	1683,43	1,13%	-0,48%	0,00821
15	1730,66	1757,71	1749,78	1,10%	-0,45%	0,00796
16	1797,43	1823,66	1815,98	1,03%	-0,42%	0,00744
17	1863,52	1888,04	1880,91	0,93%	-0,38%	0,00672
18	1927,68	1950,99	1942,83	0,79%	-0,42%	0,00610
19	1980,64	1996,91	1992,16	0,58%	-0,24%	0,00420
20	2047,04	2047,04	2047,04	0,00%	0,00%	0,00000

Table 2. Reserves mortality profile

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