Fair valuation schemes for life annuity contracts

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Abstract. The paper focuses on the fair valuation of the stochastic reserve of a life policy portfolio. The method, presented for life annuities because of their particular importance in the life insurance market, substantially fits any kind of life policy portfolio. The quantitative approach starts from regulatory and managerial outlines aimed to indicate the reserve quantification as a mark-to-market valuation of the outstanding liabilities. Numerical examples clarify the valuation scheme, comparing the current values of projected cash-flows and the corresponding ones calculated at the contractual rate.

Keywords: Life insurance, Reserve, Fair valuation, Financial risk, Demographic risk.

1 Introduction

The international accounting standards for insurance have been partly defined during 2004 and are partly in course of definitive settlement after revision. Life insurance companies in EU have to follow the new standards and consequently the consolidated financial statements will have to be drawn up in conformity with them (cf. [Jorgensen, 2004]).

The basic idea emerging from the new instructions is to depict the firm as much as possible in its realistic economic profile. In particular, as regards the solvency assessment, the guidance takes shape in the request of the reserve quantification as a mark-to-market valuation of the outstanding liabilities, the so called *fair value*. Several definitions of fair value have been proposed, the strongest line converging toward *"the market value, if a sufficiently active market exists, or an estimated market value otherwise"* (cf. [CAS, 2002]) and the following ([FASB, 2004]): *"the price at which an asset or a liability could be exchanged in a current transaction between knowledgeable unrelated willing persons"*.

It's evident that the traditional priciple basing the accounting system on historical cost is now substituted by a new standard founded on current values. The evolution of the international accounting standards reveals that the insurance business is *inside* the market and no more "preferred" rules will make possible to write out no troubled balance sheets.

The valuation techniques established in [FASB, 2004] are classified in a fair value hierarchy, in which the strenght of the connection of insurance cash flows to products traded in active markets is the ordering criterium.

As a consequence, it becomes necessary at the same time to satisfy the fair value principle, imposing the use of the market inputs as much as possible, and to overcome the lack of financial product identical to insurance assets and liabilities traded in a market.

Great importance Buhlmann's idea assumes in this question (cf. [Buhlmann, 2002]): he proposes to measure the liabilities of the insurer resulting from a single policy or a portfolio of policies, as a portfolio of financial instruments, so introducing the Valuation Portfolio (VaPo). The tool originates by the question of the stochastic discounting factor characterization, usable to price securities in arbitrage free markets, analysed by Long (cf. [Long,1990]): in that paper the author introduces the *numeraire portfolio*, defining it as a self financing trading strategy with positive value such that, if prices and cash flows are expressed in its units, the current net cash flow prices are the best prediction of the next period cash-flow prices. Buhlmann observes that the simple circumstance that the insurer sells the insurance contract or the portfolio of insurance contracts, involves that the financial instruments composing the VaPo exist in the economic reality, even if not traded on an existing market, in this way posing in evidence the character of "fair valuation" of the procedure (cf. [Buhlmann, 2004]). The financial component in Buhlmann's valuation process is properly faced by a numeraire approach, considering the cash flow generated by the policy or the portfolio of policies as expressed in "units" of Zero Coupon Bonds, since this methodology makes the valuations comparable each other.

Therefore, the current valuations, if connected with relatively simple insurance liabilities, can be estimated using prices for similar liabilities traded in active markets, in this case being in part of *level two* of the hierarchy proposed by [FASB, 2004].

The case we want to focus refers to a portfolio of life annuities, the interest in this kind of policies being due to their diffusion in the general life insurance outline and in the theoretical implications they have in the pension field. Moreover, this contractual case turns up characterised by a composite risk identity, being affected by the longevity risk, besides the technical (mortality) risk component and the financial risk one (cf. [Coppola *et al.*, 2002]).

In this paper we study the valuation of the stochastic reserve in the case of a portfolio of life annuities by means of a stochastic pricing model based on the no-arbitrage principle applied to the cash flow structure of future assets and liabilities. The valuation technique for estimating fair value will consist in expected valuation connected to cash flows discounted taking in account the systematic financial risk (cf. [FASB, 2004]) together with the demographic risk in its two displays, the accidental and the systematic components. In particular, to capture the effects of the systematic component due to the betterment of the mortality trend, we will use an opportune projected mortality table.

A comparison between a *fair valuation* and a classical procedure based on fixed valuation rates is presented.

2 The valuation scheme

Let us introduce two probability spaces $(\Omega, \mathfrak{F}', P')$, $(\Omega, \mathfrak{F}'', P'')$, where $\mathfrak{F}$' and $\mathfrak{F}$" are the σ-algebras containing, respectively, the *financial events* and the *life duration events* (referring to the unsystematic aspect of mortality). We assume that the randomness in mortality is independent on the fluctuations of interest rates. Let us consider the probability space $(\Omega, \mathfrak{F}, P)$ generated by the preceding two by means of canonical procedures; $\mathfrak{F}$ contains the information flow about mortality and financial history, represented by the filtration $\{\mathfrak{F}_k\} \subset \mathfrak{F}$, with $\mathfrak{F}_k = \mathfrak{F}'_k \cup \mathfrak{F}''_k$ and $\{\mathfrak{F}'_k\} \subset \mathfrak{F}'$, $\{\mathfrak{F}_k\}'' \subset \mathfrak{F}''$. Let us denote by N_j the number of claims (survivors or dieds according to the kind of life contract) at time j within a portfolio of identical policies. We are interested in evaluating at time t the stochastic stream of cash-flows $\hat{\mathbf{X}}^{\mathbf{t}} = N_{t+1}X_{t+1}, N_{t+2}X_{t+2}, \ldots, N_n X_n$, that is the stochastic loss at time t, referring to a portfolio perspective. In a *fair valuation* framework, we assume a frictionless market with continuous trading, no restrictions on borrowing or short-sales, the zero-bonds and the stocks are both infinitely divisible. In a risk-neutral valuation, the fair value at time t is given by

$$\mathfrak{V}_t = \mathbb{E}\left[\sum_{j>t} N_j X_j v(t,j) | \mathfrak{F}_t\right] \tag{1}$$

where $v(t,j)$ is the present value at time t of one monetary unit due at time j, and $\mathbb{E}$ represents the expectation under the risk-neutral probability measure, whose existence derives by well known results, based on the completeness of the market. For a deeper understanding, it is necessary to remark that the demographic valuation is not supported by the hypotesis of the completeness of the market. In any case it is possible to introduce an appropriate probability measure, as suggested in [De Felice and Moriconi, 2004].

Equivalently, indicating by c the number of policies at time 0, in the specific case of surviving benefits, we can write

$$\mathfrak{V}_t = \mathbb{E}\left[\sum_{j>t} c\mathbf{1}_{\{K_{x,t}>j\}} X_j v(t,j) | \mathfrak{F}_t\right] \tag{2}$$

where the indicator function $\mathbf{1}_{\{K_{x,t}>j\}}$ takes the value 1 if the curtate future lifetime of the insured, aged x at issue, takes values greater than $t+j$ ($j = 1, 2, \ldots$), that is if the insured aged $x+t$ survives up to the time $t+j$, 0 otherwise. By virtue of the basic assumptions on the risk sources, we get

$$\mathfrak{V}_t = \sum_{j>t} cX_j \mathbb{E}[\mathbf{1}_{\{K_{x,t}>j\}}|\mathfrak{F}_t]\mathbb{E}[v(t,j)|\mathfrak{F}_t] \tag{3}$$

$$= \sum_{j>t} cX_j \ {}_tp_x \ {}_jp_{x+t}\mathbb{E}[v(t,j)|\mathfrak{F}_t]$$

where ${}_kp_y$ denotes the probability that an insured aged y survives until the age $y+k$. The terms on the right hand side clearly show that the expected discounted value of the stochastic stream can be regarded as the valuation of a portfolio of zero coupon bonds with maturities in j. The price in t of such portfolio, in a fair valuation approach, can be regarded as the market price of the zero coupon bonds portfolio, and therefore the current value of it.

In order to provide a more concrete application, we consider a portfolio of c insureds aged x, each of whom having a deferred life annuity policy with premiums payable at the beginning of the first T years and benefits payable at the beginning of each year after T if the insured is alive.

According to the notations in [Cocozza *et al.*, 2004], we assume

- B_s=benefit payable to each insured at time s,
- P_s=premium payable by each insured at time s,
- $\hat{\mathbf{X}}^{\mathbf{s}}$= the flow at time s related to each insured, with the generic element represented by the following scheme:

$$X_s = \begin{cases} -P_s & \text{if } s < T \\ B_s & \text{if } s \geq T \end{cases}$$

According to Buhlmann, we'll valuate the financial component of the risk neutral expected value accordingly with a numeraire approach: we determine the value of each flow using a market based discount factor, expressing the current price of the default free unit discount bond issued at time t and maturing at time j ($j \geq t$) .

A representation of the portfolio of financial instruments at time 0 we refer to, that is the Valuation Portfolio, is here reported, having indicated by $Z(t,j)$ the Zero Coupon Bond issued at time t and maturing at time j and considering constant premiums and benefits

$$(VaPo)_0 = \begin{cases} \text{unit} & \text{number of units} \\ Z(0,0) & -cP \\ Z(0,1) & -N_1 P \\ \dots & \dots \\ Z(0,T-1) & -N_{T-1}P \\ Z(0,T) & N_T B \\ \dots & \dots \end{cases}$$

while the generic element of the $(VaPo)_t$ results:

$$Z(t,j) = \begin{cases} -N_{t+j/t}P & \text{if } j < T \\ N_{t+j/t}B & \text{if } j \geq T \end{cases}$$

with $N_{t+j/t}$ the number of survivors at time $t+j$ belonging to the group of those, among the c initial insured at time 0, are living at time t.

The calculations in (3) can be replicated also for a non-homogeneous portfolio of life annuities. In fact (cf. [Parker, 1997]) in this case we can divide the porfolio in homogeneous sub-portfolios, say m their number, identified by common characteristics, such as age at issue, policy duration, and so on. Let us assume

- n_i = policy duration for the i-th group
- c_i= number of policies in the i-th group ($\sum_{i=1}^{n} = c$)
- x_i = age at issue of the insureds of the i-th group
- $\hat{\mathbf{X}}^{\mathbf{i,j}}$= the stochastic flow at time j related to the i-th group
- $n = \max_i \; n_i$
- $N_{i,j}$ = number of survivors in the i-th group at time j

Now we can write the value in t for the entire portfolio

$$\mathfrak{V}_t = \mathbb{E}\left[\sum_{i=1}^{m}\sum_{j>t} N_{i,j}X_{i,j}v(t,j)|\mathfrak{F}_t\right] = \tag{4}$$

$$= \mathbb{E}\left[\sum_{i=1}^{m}\sum_{j>t} X_{i,j}c_i\mathbf{1}_{\{K_{x_i,t}>j\}}v(t,j)|\mathfrak{F}_t\right] =$$

$$= \sum_{i=1}^{m}\sum_{j>t} c_i X_{i,j}\; {}_tp_{x_i}\; {}_jp_{x_i+t}\mathbb{E}[v(t,j)|\mathfrak{F}_t]$$

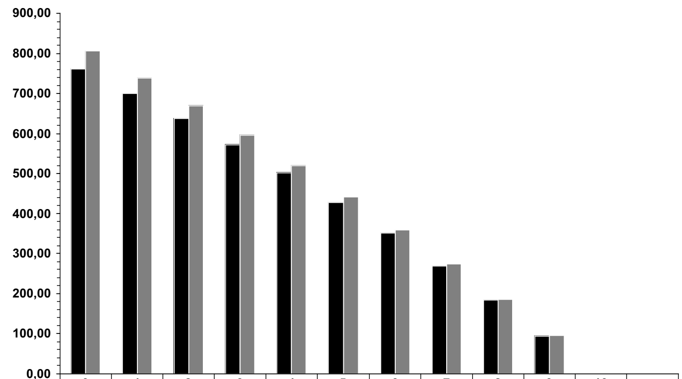
with obvious meaning of the symbol $\mathfrak{F}_t$ in this context.

3 Applications

We present an application of formula (3) for the case of an homogeneus portfolio of 100 immediate 10-years temporary unitary annuities, for policyholders aged 40 at issue. We assume a term structure of interest rates based on the Cox-Ingersoll-Ross square root process

$$dr_t = -k(r_t - \gamma)dt + \sigma\sqrt{r_t}dB_t \tag{5}$$

with k and σ positive constants, γ the long term mean and B_t a Brownian motion. In Fig.1 we report the fair values of the reserves and compare them with the corresponding values calculated at the contractual rate 0.04. In particular we assume for the CIR process γ=0,0452, σ=0,0053 and the initial value $r_0 = 0,0279$ (cf. [Cocozza *et al.*, 2004]). We use the survival probabilities deduced by the Italian Male Mortality called RG48, which take into account also the phenomenon concerning the improvement in the mortality trend.

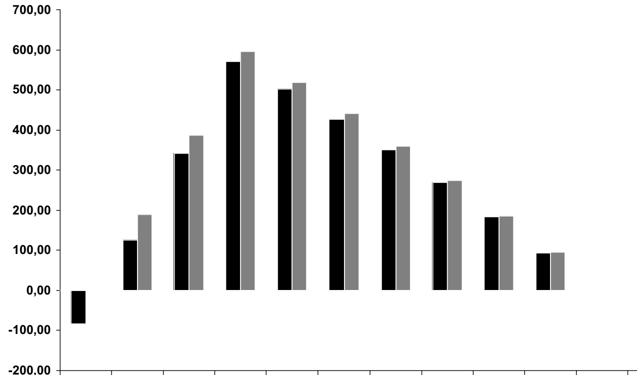


	0	1	2	3	4	5	6	7	8	9	10	
Cir based	760,868	700,995	637,881	571,392	501,391	427,744	350,313	268,964	183,557	93,9507	0	
Fixed 4%	806,081	738,415	668,143	595,165	519,386	440,704	359,017	274,22	186,199	94,8351	0	

Evaluation time (years)

Fig. 1. Reserves of life annuities (t=0,1,. . . ,10)

In Fig.2 we present the results obtained, under the above hypotheses about survival and rates, for a portfolio of deferred (T=3 years) life annuities with the same characteristics mentioned above, but periodic premiums are paid at the beginning of each year of the deferment period. In $t = 0$ we have considered also the first premium paid. Since the premiums are calculated at 4%, in $t = 0$ the equity condition is obviously realized only for the contractual rate.



	0	1	2	3	4	5	6	7	8	9	10	
Cir based	-83,74	124,457	342,741	571,392	501,391	427,744	350,313	268,964	183,557	93,9507	0	
Fixed 4%	0	189,352	387,673	595,165	519,386	440,704	359,017	274,22	186,199	94,8351	0	

Evaluation time (years)

Fig. 2. Reserves of deferred life annuities (t=0,1,. . . ,10)

We can observe that the current values of the stochastic loss are always smaller than the corresponding calculated at the contractual rate; these differences decrease when the evaluation time increases.

This phenomenon is due to the relation between the impact on the reserve of the variations of the interest rate and the residual time of the policies (cf. [Cocozza *et al.*, 2004]). The above factors directly influence the reserve variation: it depends on the reserve amount and the reserve duration, and both these parameters decrease when the policy residual time decreases, so the impact of the fluctuations of the interest rates are stronger at the beginning of the evaluation time interval.

4 Concluding remarks

According to the general guide-lines provided by the international institutions, the reserve quantification will consist in a mark-to-market valuation of the outstanding liabilities, or, in other words, in its *fair value*. In this framework in the paper the fair value of the losses of a life annuity portfolio is analysed, on the basis of a risk-neutral valuation. The new appraisal will be beneficial to the transparency and for giving the actual economic profile of a life insurance business. More and clearer information will reach the policy holders and the investors, and a higher degree of comparison will be in force among EU life insurance companies (cf [Jorgensen 2004]).

From the point of view of actuarial valuations, the strong aspect of the question will disclose in a bigger volatility of the results, certainly very much greater than that ones gushing from the traditional historical cost based method.

A further development will be a sensitivity analysis concerning the parameters connected to the market fluctuations; moreover it could be interesting to implement simulation procedures aimed to determine the distribution of the stochastic loss as the evaluation time varies and this would provide useful information about solvency assessment.

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