Identification, estimation and control of uncertain dynamic systems: a nonparametric approach

Nadine Hilgert[1], Vivien Rossi[1,2], and Jean-Pierre Vila[1]

[1] UMR Analyse des Systèmes et Biométrie,
ENSA.M - INRA, 2 place Viala, Bât. 29
34060 Montpellier Cedex 1, France
(e-mail: hilgert@ensam.inra.fr, vila@ensam.inra.fr)

[2] I3M, UMR CNRS 5149, Université Montpellier 2,
Case Courrier 051, Place Eugène Bataillon
34095 Montpellier Cedex 5, France
(e-mail: rossiv@ensam.inra.fr)

Abstract. This paper is devoted to a short presentation of the use we did of nonparametric estimation theory for the estimation, filtering and control of uncertain dynamic systems. The fundamental advantage of this approach is its low dependence from any a priori modeling assumptions about uncertain dynamic components. It appears to be of great interest for the control of general discrete-time processes, and in particular biotechnological processes, which are emblematic of nonlinear uncertain and partially observed systems.
Keywords: Discrete-time stochastic systems, Markov controlled processes, Nonparametric identification, Predictive control, Nonlinear filtering, Fault detection.

1 Introduction

This paper is devoted to a survey of the use of nonparametric estimation theory for the estimation, filtering and control of uncertain dynamic systems. It relies on a set of works we have been developing for more than ten years and which emphasizes the efficiency of these nonparametric tools in functional estimation as well as in probability density estimation.

The frame of these developments is that of the control of general discrete-time processes, and in particular biotechnological processes, which are emblematic of nonlinear uncertain and partially observed systems. The field of bioprocess modeling and control offers typical examples of structural time-variations problems which cannot be handled by classic control methods: the dependence of the kinetic coefficients on biomass and substrate state variables is affected by functional fluctuations and not merely parametric ones. In that case, a more appropriate approach would be robust control, in which uncertainty is explicitly accounted for at the beginning of the control design through numerical or functional bounds. However, the performance of the related controllers can be sensitive to settings that are too much conservative or too much optimistic. The nonparametric approach is free from these prior assumptions: through a stochastic learning process, uncertain functional components are progressively and automatically estimated as deterministic or random functions of the measured quantities, in accordance with their actual but unknown and possibly time-varying structures. The use of this functional estimation procedure, compared with the usual and more or less arbitrary choice of these model components, contributes to the reduction of one source of model inadequacy. Moreover, the stochastic frame in which these nonparametric models are designed allows some uncontrolled disturbances such as measurement errors and parameter variations to be accounted for.

In the following we shall present successively application of this nonparametric approach to identification, filtering and control of dynamic systems.

2 Identification and estimation of nonlinear stochastic processes

The uncertain processes under consideration belong to the general class of controlled Markov chains.

They are represented by discrete-time autoregressive models of the following type:

$$X_{t+1} = F_t(X_t, U_t, \varepsilon_{t+1}), \tag{1}$$

where $X_t \in \mathbb{R}^s$, $U_t \in \mathbb{R}^m$ and ε_t are the output, input and noise of the system, respectively. Driving function F_t may be completely or partly unknown, according to the degree of uncertainty in the analytical knowledge of the process. This function may be deterministic or stochastic and is supposed to obey some regularity conditions (see §2.1). Moreover, when the state variable X_t is not observed, an observation model is supposed to be available, of the general form

$$Y_t = G_t(X_t, U_t, \eta_t) \tag{2}$$

where $Y_t \in \mathbb{R}^q$ and G_t is a known function and η_t an observation noise.

Estimating function F_t in model (1) may be intricate. The following particular case with an additive noise is more frequently met in practice:

$$X_{t+1} = f_t(X_t, U_t) + \varepsilon_{t+1}, \tag{3}$$

in which function f_t, from $\mathbb{R}^s \times \mathbb{R}^m$ to $\mathbb{R}^s$, may be completely or partly unknown. We are specifically interested in a type of non-linear models where the control variable U_n acts in a known part of function f_t. They are models of the field of bioprocess modeling and control, and are of form:

$$X_{t+1} = A_t(X_t)g_t(X_t) + B_t(X_t, U_t) + \varepsilon_{t+1}, \tag{4}$$

where A_t and B_t are known functions and g_t is unknown. Function g_t is for example the growth rate of some microorganism population whose concentration is a component of the state variable X_t. The control variable U_t is for example a dilution rate of a polluted water into a bioreactor.

Other examples of model (3) are for instance the evolution models of bacteria populations in food under the influence of environment covariates (U_t), or, in another field, models that describe the position of a space craft under control.

The following subsection is dedicated to the identification of model (3) when unknown (or partially unknown), with state X_t completely observed. The well-known convolution kernel method is applied to estimate function f_t (or only a subpart of it).

In subsection 2.2 state variables X_t are not supposed to be observed anymore and the issue considered is now that of their estimation, *i.e.* filtering, from knowledge of the observed variables Y_t and assuming knowledge of model F_t.

2.1 Identification of the model with convolution kernel estimators

Kernel smoothing methods are among the most reknown nonparametric estimation and prediction methods. They belong to the family of smoothing methods (orthogonal polynomials, splines,...) and are based on a local averaging procedure. They are widely used to estimate probability density functions and regression functions, see [Bosq, 1996].

When the whole function f_t is unknown in model (3), we can consider the following recursive kernel estimator, for all $x \in \mathbb{R}^s$ and $u \in \mathbb{R}^m$:

$$\widehat{f}_t(x,u) = \frac{\sum_{i=0}^{t-1} \delta_{1,i}^{-s}\delta_{2,i}^{-m} K_1(\frac{x-X_i}{\delta_{1,i}})K_2(\frac{u-U_i}{\delta_{2,i}})X_{i+1}}{\sum_{i=0}^{t-1} \delta_{1,i}^{-s}\delta_{2,i}^{-m} K_1(\frac{x-X_i}{\delta_{1,i}})K_2(\frac{u-U_i}{\delta_{2,i}})}, \tag{5}$$

The functions K_1 and K_2 are two kernel functions. They are real positive symmetric functions integrating to one.
The sequences $(\delta_{1,i})$ and $(\delta_{2,i})$, called the bandwidths, have to be positive and decreasing. See [Georgiev, 1984] for the case of an *i.i.d.* sequence (U_t), and [Wagner and Vila, 2001] for a more general situation.

In the case of biotechnological processes, the partially known model (4) is the most frequently met. In that case, the kernel estimation of g_t is given by:

$$\widehat{g}_t(x) = \frac{\sum_{i=0}^{t-1} \delta_i^{-s} K(\frac{x-X_i}{\delta_i})A_i^{-}(X_i)(X_{i+1} - B_i(X_i,U_i))}{\sum_{i=0}^{t-1} \delta_i^{-s} K(\frac{x-X_i}{\delta_i})}. \tag{6}$$

for all $x \in \mathbb{R}^s$. A_i^- is a general inverse of matrix A_i and K is the kernel function and (δ_i) the bandwidth.

The statistical convergence properties of kernel estimators (5) or (6) have been established under various assumptions about

- the probability distribution of the noise ε,
- the existence of admissible control strategies $(U_t)_{t\geq 1}$ able to stabilize the model (X_t)
- the behaviour of the unknown set of stochastic functions f_t (respectively g_t), which must be quite "stable", corresponding to a convergent sequence f_t (resp. g_t) or an *i.i.d.* functional sequence f_t (resp. g_t).

As regard the bandwidth parameters, the form $\delta_i = \gamma i^{-\alpha}$ is one for which convergence results have been established [Duflo, 1997], [Portier and Oulidi, 2000], [Hilgert *et al.*, 2000]. In some cases, an optimal choice of the bandwidth parameters can be determined by cross validation procedures, see [Vieu, 1991] for instance. From a theoretical point of view, we may distinguish between

- the *a.s.* uniform convergence on compact sets, which requires kernel functions with compact support, as the Epanechnikov kernel for example.
- the stronger *a.s.* convergence on dilated compact sets, which requires positive kernel functions, as the Gaussian kernel for example.

2.2 Estimation of state variables with convolution particle filters

Besides its efficiency in functional estimation of uncertain models as seen in the previous section, the nonparametric approach as proved to be useful as well in probability density estimation of unobserved state variables, *i.e.* in filtering problems.

The objective is now to estimate the unobserved state variable X_t from the analytical knowledge of state model F_t (1) and the observed variables $Y_{1:t} = (Y_1, \cdots, Y_t)$. When F_t and G_t correspond to linear functions of X_t and U_t with additive noises, the well-known Kalman filter provides an optimal estimate of the probability distribution of X_t conditionally to $Y_{1:t}$, $P(X_t|Y_{1:t})$. In the other cases, only the so-called Monte Carlo filters or particle filters (see [Doucet *et al.*, 2001] or [Del Moral, 2004]) provide consistent estimates of $P(X_t|Y_{1:t})$. The main principle of these filters is to build an estimate of $P(X_t|Y_{1:t})$ through the simulation of a large number N of random state particles $\{x_i\}$ which are then weighted according to their likelihoods with respect to the observed variables up to time t.

However the usual particle filters require, in practice, the function G_t to be additive in the observation noise η_t, and the analytic form of the density of η_t to be known.

This last assumption really reduces the applicative potential of these particle filters. The convolution particle filters we proposed in [Rossi, 2004] and [Rossi and Vila, 2004] drop this assumption thanks to the use of convolution kernels to estimate the conditional density $p(X_t|Y_{1:t})$ supposed to exist. The following algorithm shows the implementation of the Resampled-Convolution Filter, one of the filters we developed [Rossi, 2004]:

Starting from a given initial probability density $p_0(X_0)$ and N simulated state values $(\tilde{X}_0^1, \ldots, \tilde{X}_0^N))\mathrm{sim} p_0(X_0)$,

At time t:

(i) Sampling Step:
$(\tilde{X}_t^1, \ldots, \tilde{X}_t^N)\mathrm{sim} p_t^N$ where p_t^N is the last estimated state conditional density.

(ii) Evolving Step: for $i = 1..N$, $(\tilde{X}_t^i) \longrightarrow (\tilde{X}_{t+1}^i, \tilde{Y}_{t+1}^i)$ by simulation of model (1)-(2).

(iii) Approximation Step:
$$p_{t+1}^N(X_{t+1}|Y_{1:t+1}) = \frac{\sum_{i=1}^N K_{2,\delta_N}(Y_{t+1} - \tilde{Y}_{t+1}^i) K_{1,\delta_N}(X_{t+1} - \tilde{X}_{t+1}^i)}{\sum_{i=1}^N K_{2,\delta_N}(Y_{t+1} - \tilde{Y}_{t+1}^i)}$$
with $K_{1,\delta_N}(x) = \delta_N^{-s} K_1\left(\frac{x}{\delta_N}\right)$, $x \in \mathbb{R}^s$ and $K_{\delta_N}(y) = \delta_N^{-q} K_2\left(\frac{y}{\delta_N}\right)$, $y \in \mathbb{R}^q$.

This algorithm ensures to get an "on line" L_1-convergent estimate of the density $p_t(X_t|Y_{1:t})$ when the particles number N tends to infinity ([Rossi, 2004] or [Rossi and Vila, 2004]).

3 Nonparametric adaptive and predictive control

The objective considered in this section is to find a control sequence $(U_t)_{t\geq 1}$ which forces the state variables $(X_t)_{t\geq 1}$, to follow as best as possible a given bounded trajectory $(X_t^*)_{t\geq 1}$. The state variable X_t is now again supposed to be observed and to evolve according to model (3), with function f_t completely or partly unknown.

Two control strategies are considered in the following according to the immediate or anticipative trajectory fitness considered, the second one being furthermore a generalization of the first.

3.1 Adaptive tracking control

Consider the particular case of model (4) particularly convenient for the biotechnological systems, in which g_t is unknown. An adaptive control strategy has to be built from the nonparametric estimate (6), which ensures the stochastic closed-loop stability. This last property is indeed necessary to ensure the convergence properties of the kernel estimator $\widehat{g}_t$. When B_t is supposed to be invertible with respect to U_t, let us consider a solution U_t such that

$$B_t(X_t, U_t) = X_{t+1}^* - A_t(X_t)\widehat{g}_t(X_t)\mathbb{1}_{E_t}(X_t) - A_t(X_t)g^*(X_t)\mathbb{1}_{E_t^c}(X_t)$$

where E_t is a subset of the state space, depending on the kernel estimate $\widehat{g}_t$ and on g^*, an a priori knowledge of g_t.

It has been shown that this strategy is asymptotically optimal:

$$\frac{1}{t}\sum_{i=1}^{t} \|X_i - X_i^*\|^2 \xrightarrow{a.s.} trace(\Gamma) \quad \text{as } t \to \infty,$$

where Γ denotes the covariance matrix of the noise ε_t.

See [Portier and Oulidi, 2000] and [Hilgert, 1997] for more details.

3.2 Optimal predictive control

Let us consider again state model (3) with unknown function f_t and still the assumption of observed X_t.

The principle of the so-called predictive control is now well-known among control theorists (see for example [Camacho and Bordons, 1995]). The specificity of predictive control is to consider the future values to be followed by the state system in a near forward horizon of given length H. More precisely at each time step the future values of the state variables on the horizon are predicted conditionally to intermediary control values. These control values are then optimized in order to minimize some discrepancy function between the predicted state values and that of the trajectory on the same horizon. The first of these optimal values of the control variable is then applied to the system which enters then the following time step and the predictive horizon is translated. Such an anticipating strategy confers to predictive control a significant advantage among on-line tracking control strategies, and is particularly adapted to the control of processes with slow dynamic such as the biotechnological processes. The main question raised by the predictive control algorithms is that of the stability of the closed loop. For deterministic systems several constraint conditions have been designed to ensure this stability (see [Mayne *et al.*, 2000] for a recent survey). For stochastic system this issue is still open for the general case. We consider it in the nonparametric approach to follow and solve it in a simple case.

A nonparametric predictive control algorithm for uncertain system:
At step t,

- let
$$J_t = \sum_{j=1}^{j=H} \| X_{t+j}^* - f_{t+j-1}^j \left(u^1, \dots, u^j \,|\, X_{i,\, i\leq t} \,;\, U_{i,\, i\leq t-1}\right) \|^2$$
where
  - H is the chosen length of the receding horizon
  - $\widehat{X}_{t+j} = f_{t+j-1}^j \left(u^1, \dots, u^j \mid X_{i,\, i\leq t} \;;\; U_{i,\, i\leq t-1}\right)$ is a consistent estimate to be looked for $\mathrm{E}\left[X_{t+j} \,|\, X_{i,\ i\leq t} \;;\; U_{i,\ i\leq t-1} \;;\; U_t = u^1, \dots, U_{t+j-1} = u^j\right]$ which is itself the minimum variance predictor of the state value X_{t+j}.

- Find
$$\begin{aligned}\bar{U}_t &= (U_t^1, \ldots, U_t^H) \\ &= \mathrm{argmin}_{\|u^1\| \leq M, \ldots, \|u^H\| \leq M} \; J_t\end{aligned}$$
  with M: upper bound constraint in the control values.
- take $U_t = U_t^1$
- $t = t + 1$

A j-step-ahead nonparametric state predictor:
Let $Z_t^j = (X_t, U_t, \ldots, U_{t+j-1})^t$
Let us consider as estimate of $\mathrm{E}(X_{t+j} \mid Z_t^j = z)$

$$\widehat{X}_{t+j} = \widehat{\mathrm{E}}(X_{t+j} \mid Z_t^j = z) = \frac{\sum_{t=1}^{t-j} |\det(\delta_t^{-1})| K\left(\frac{z - Z_t^j}{h_t}\right) X_{t+j}}{\sum_{t=1}^{t-j} |\det(\delta_t^{-1})| K\left(\frac{z - Z_t^j}{h_t}\right)}$$

where K is a kernel of dimension $(s + jm)$ and the matrix δ_t, of same dimension, is the bandwidth parameter of K.
For uncontrolled process, the asymptotic behaviour of $\widehat{X}_{t+j}$ has been characterized under mixing conditions and stationarity assumptions [Bosq, 1996]. These results are not applicable for the controlled processes we consider in this paper since the applied control values are state dependent. However for the simplest case, $H = 1$, stability of the closed loop, almost sure uniform dilated convergence of the kernel predictor and suboptimality of the control strategy has been established under regular conditions ([Wagner, 2001], [Wagner and Vila, 2001]) in both cases of interest for the f_t sequence (see section 2.1).

Remark 1: the minimization of the criterion function J_t at step t with respect to the constrained control variables $(u^1, \cdots, u^H)$, can be done by standard descent algorithm. We developed also a more efficient neural network-based minimization procedure and applied it online on a real biotechnological depollution process [Vila and Wagner, 2003].

Remark 2: the choice of the length of the predictive horizon H must result from a case by case compromise between long term optimality of the predictive control (high values for H) and the quality of the kernel predictors (low values).

4 Conclusion and perspectives: towards the nonparametric supervision of uncertain systems

When dealing with process control, an unavoidable issue is that of supervision. Supervision consists in being able to detect any default in the process (*e.g.* pump clogging in a bioprocess), locating the default and remedying it (by an appropriate sequence of actions). From a statistical point of view, the problems of detection and isolation of a default are equivalent to detecting abrupt changes in a stochastic process, and testing multiple hypotheses to determine the faulty scenario among a number of possible scenarii of defaults [Dubuisson, 2001].

There exist many statistical procedures to answer such questions, see [Basseville and Nikiforov, 1993]. A well-known one is the CuSum test. It is based on a comparison, at each time instant, of the difference between the log-likelihood ratio value and its current minimal value, with respect to a fixed threshold. Most of these techniques require knowledge of both state and observation models.

When the state model is uncertain, the question is still open. However combining nonparametric estimates as (5) or (6) with classical test procedures gave us encouraging results on real experimental data issued from a depollution process.

Moreover, introducing filtering methods such as the one proposed above, will allow to generalize these nonparametric detection procedures to the most frequent situation of indirectly observed systems described by models (1) and (2).

References

[Basseville and Nikiforov, 1993]M. Basseville and I.V. Nikiforov. *Detection of abrupt changes - Theory and application*. Prentice Hall, 1993.

[Bosq, 1996]D. Bosq. *Nonparametric Statistics for Stochastic Processes, Estimation and Prediction*. Springer-Verlag, New York, 1996.

[Camacho and Bordons, 1995]E.F. Camacho and C. Bordons. *Model Predictive Control in the Process Industry*. Springer-Verlag, 1995.

[Del Moral, 2004]P. Del Moral. *Feynman-Kac Formulae. Genealogical and Interacting Particle Systems with Applications*. Springer, 2004.

[Doucet *et al.*, 2001]A. Doucet, N. De Freitas, and N. Gordon. *Sequential Monte Carlo Methods in Practice*. Springer, 2001.

[Dubuisson, 2001]B. Dubuisson. *Automatique et statistiques pour le diagnostic*. Hermes Science Europe Ltd, 2001.

[Duflo, 1997]M. Duflo. *Random Iterative Models*. Springer-Verlag, New York, 1997.

[Georgiev, 1984]A. Georgiev. Nonpararametric system identification by kernel methods. *IEEE Trans. Automat. Control*, pages 356–358, 1984.

[Hilgert *et al.*, 2000]N. Hilgert, R. Senoussi, and J.-P. Vila. Nonparametric identification of controlled nonlinear time varying processes. *SIAM J. Cont. Optim.*, pages 950–960, 2000.

[Hilgert, 1997]N. Hilgert. *Identification et contrôle de processus autorégressifs non linéaires incertains : application á des procédés biotechnologiques*. PhD Thesis, Paris XI, 1997.

[Mayne *et al.*, 2000]D.Q. Mayne, J.B. Rawlings, C.V. Rao, and P.O. Scokaert. Constrained model predictive control: stability and optimality. *Automatica*, pages 789–814, 2000.

[Portier and Oulidi, 2000]B. Portier and A. Oulidi. Nonparametric estimation and adaptive control of functional autoregressive models. *SIAM J. Cont. Optim.*, pages 411–432, 2000.

[Rossi and Vila, 2004]V. Rossi and J.-P. Vila. Nonlinear filtering in discrete time : A particle convolution approach. *Technical Report 04-03, Groupe biostatisque ENSAM/INRA/UM2 Montpellier*, 2004.

[Rossi, 2004]V. Rossi. *Filtrage Non Linéaire par Noyaux de Convolution. Application à un Procédé de Dépollution Biologique.* PhD Thesis, ENSA Montpellier, 2004.

[Vieu, 1991]P. Vieu. Nonparametric regression: optimal local bandwidth choice. *J. R. Statist. Soc. B*, pages 453–464, 1991.

[Vila and Wagner, 2003]J.-P. Vila and V. Wagner. Predictive neuro-control of uncertain systems: design and use of a neuro-optimizer. *Automatica*, pages 767–777, 2003.

[Wagner and Vila, 2001]V. Wagner and J.-P. Vila. Estimation non paramétrique et contrôle prédictif neuronal de processus autorégressifs non linéaires incertains. *Journées de Statistique de la SFdS*, pages 711–714, 2001.

[Wagner, 2001]V. Wagner. *Identification non paramétrique et contrôle prédictif neuronal de processus non linéaires incertains de type biotechnologique : application à un procédé de dépollution biologique.* PhD Thesis, ENSA Montpellier, 2001.