

# A New Modeling Approach Investigating the Diffusion Speed of Mobile Telecommunication Services in EU-15

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**Abstract.** The objective of this paper is to investigate the impact of the time-delay effect on the diffusion of mobile telecommunication services in EU. It has been proved from several studies that the time-delay between the awareness and the adoption phase of mobile services-potential users determines the speed of the mobile telecommunication service diffusion and can be used effectively for ranking or cluster purposes in cases when the diffusion of a new product in different countries is studied. The proposed modeling approach originates from the well-known logistic model where it is assumed that the ordinary contagion process does not take place instantly but after some certain amount of time. A proper modification of the proposed model described by a time lag ordinary differential equation can be solved analytically and its properties for several parameters' combination are investigated. Moreover, a new diffusion speed index is proposed and the correlation between the time-delay index and the proposed diffusion speed index is examined. Finally the model is applied to real data concerning the mobile services diffusion in fifteen counties of EU from 1990 to 2002. Based on the estimated parameters of the model produced for each country a ranking and a clustering of the EU countries based on their derived diffusion speed and time-delay indexes are provided.

**Keywords:** Time-delay model, Diffusion speed, Innovation diffusion modeling, Technology marketing-management, Modeling Telecommunication services.

## 1 Introduction

Today, forecasting technology in economic activity is no more avoidable than in forecasting weather in daily life. In fact, voluminous literature has explored different growth-curve models in forecasting the diffusion process of new technologies. Early contributions to this subject are attributed to scientists who noted the analogy between the epidemic process and the social adoption process [Griliches, 1957], [Mansfield, 1961], [Bass, 1969], [Fisher and Pry, 1971], [Blackman, 1972], [Sharif and Kabir, 1976], [Sharif and Ramathan, 1984]. They came to a general agreement that the proportion of adopters rises at an accelerating rate during the early stages of the diffusion process and then at a declining rate until the population of potential adopters

has been exhausted. Later, economists and technologists joined the field to predict the marketability of new products, trajectory of technology process, penetration rate of the advanced manufacturing technologies ([Skiadas, 1985], [Skiadas, 1986], [Skiadas, 1987], [Kumar and Kumar, 1992] and [Mead and Islam, 1998]). Initiated with a simple logistic function, various curves have been empirically derived to investigate the patterns of technological growth process. These curves differ from one another in terms of number of parameters, the point of inflection, the symmetric or non-symmetric shape of their shape, etc.

This paper is using an earlier developed approach ([Poznanski, 1983], [Skiadas, 1986]) originating from the logistic innovation diffusion model which incorporates the time-delay between the awareness and the adoption phase during the classical contagion process between the adopters and the potential adopters of a new technology. It has been proven that the time-delay affects the performance of a new technology launching and speed and it can be used for comparison purposes, in order to study the innovation diffusion among groups of potential adopters with different characteristics. The proposed model can be solved analytically and presents very attractive properties well documented in the field of innovation diffusion representation. Additionally, an expression on the relationship between the time-delay parameter and the diffusion speed is presented. The time-delay innovation diffusion model is applied to the data of mobile telephony services diffusion in EU-15, in order to determine the existence of penetration patterns in relation to the time delay between the awareness and the adoption phase of the potential adopters. Finally, the outcomes are used for a ranking of the investigated countries.

## 2 A model expressing the time-delay of adoption-diffusion process

The diffusion of an innovation in a stable and homogeneous system with no external influence is traditionally expected to follow a symmetric S-shaped pattern represented by the well known logistic curve [Griliches, 1957]. More specifically, let  $X_t$  denote the number of agents that have adopted the new technology in time  $t$ . Let  $X^*$  denote the total number of potential adopters. Then the following o.d.e expresses the dynamics of the innovation diffusion process through the contagion process between the adopters and the potential adopters:

$$\frac{dX_t}{dt} = \frac{b}{X^*} \cdot X_t \cdot (X^* - X_t) \quad (1)$$

which implies that  $b$  represents the growth rate of the numbers of adopters relative to the proportion of agents who have not yet adopted the innovation. The innovation's penetration level follows an S-shaped pattern with

maximum diffusion speed reached when half of the total number of potential adopters has adopted the new technology.

This traditional approach in defining the innovation diffusion process assumes that the process takes place in a stable and homogeneous system in which the innovation spreads without any affection of the system's structure. In such cases the diffusion follows a symmetric pattern similar to those provided by eq. (1). The symmetry is also retained in the presence of external influences (e.g. promotional activities) which are not acting directly to the system's structure. However, many studies have proven that the presence of symmetry is not the general rule in innovation diffusion process ([Mahajan *et al.*, 1961], [Skiadas, 1985], [Skiadas, 1986], [Skiadas, 1987]). In the majority of new technology penetration patterns the asymmetry is caused by several factors such as cultural status, economic conditions, demographics (population density, urbanization, and educational level), governmental policy, technology utility, technology familiarity, etc. [Bakalis *et al.*, 1997]. The incorporation of such a critical aspect of the diffusion process into the process representation efforts not only provides more flexible models but can also lead to the revelation of several interesting properties of the innovation diffusion process.

Equation (1) assumes an immediate interaction between the adopters and the potential adopters of a new product leading to a symmetric diffusion pattern. However, this assumption is not always true since there is always a time-delay between the time of interaction occurrence and the adoption time. Thus, the potential adopters  $X^* - X_t$  at time  $t$  interact with the adopters  $X_{t-T}$  at time  $t - T$ . Taking into account the above consideration, the original logistic model takes the following form:

$$\frac{dX_t}{dt} = \frac{b}{X^*} \cdot X_{t-T} \cdot (X^* - X_t) \quad (2)$$

where  $T$  is the mean value of all time-delays occurring between the adopters and the potential adopters of the technology under investigation. Equation (2) cannot be easily handled and therefore an appropriate transformation is needed in order to have an approximate solution. By applying the Taylor series expansion to the expression  $X_{t-T}$  we have:

$$X_{t-T} = X_t - T \cdot \frac{dX_t}{dt} + \frac{T^2}{2} \cdot \frac{d^2X_t}{dt^2} - \frac{T^3}{3!} \cdot \frac{d^3X_t}{dt^3} + \dots \quad (3)$$

Provided that the parameter  $T$  is not too large compared to the total time interval, the two first terms of the whs of equation (3) could be retained. Then the equation (3) can be written as:

$$X_{t-T} = X_t - T \cdot \frac{dX_t}{dt} \quad (4)$$

Introducing equation (4) into equation (2) the following delay ordinary differential equation (ODE) results:

$$\frac{dX_t}{dt} = \frac{b}{X^*} \cdot \left[ X_t - T \cdot \frac{dX_t}{dt} \right] \cdot (X^* - X_t) \quad (5)$$

The appropriate rearrangements in equation (5) yields:

$$\frac{dX_t}{dt} = \frac{b}{1 + b \cdot T} \cdot \frac{X_t \cdot (X^* - X_t)}{X^* - \frac{b \cdot T}{1 + b \cdot T} \cdot X_t} \quad (6)$$

Setting

$$b^* = \frac{b}{1 + b \cdot T} \quad (7)$$

and then

$$b^* \cdot T = 1 - \sigma \quad (8)$$

equation (6) takes the form:

$$\frac{dX_t}{dt} = b^* \cdot \frac{X_t \cdot (X^* - X_t)}{X^* - (1 - \sigma) \cdot X_t} \quad (9)$$

Equation (9), is a special case of a family of generalized innovation diffusion models proposed by [Skiadas, 1985], [Skiadas, 1986] aiming to represent the innovation diffusion process. When  $\sigma = 1$  then equation (9) results in the above described logistic model, whereas when  $\sigma = 0$  it results in the exponential model. The solution of ODE (9) has given by Skiadas (1985) and has the following form:

$$\ln(X_t) - \sigma \cdot \ln(X^* - X_t) = \ln(X_0) - \sigma \cdot \ln(X^* - X_0) + b \cdot t \quad (10)$$

where  $X_0$  represents the numbers of adopters at time  $t = 0$ . The inflection point of the above model is given by [Skiadas, 1985] and has the following form

$$X_{inf} = X^* \cdot \frac{1 - \sqrt{\sigma}}{1 - \sigma} \quad (11)$$

The inflection point is considered as measure of asymmetry in every technology diffusion case. Equation (11) reveals that the proposed model is very flexible since the inflection point takes values from 0 to  $X^*$  depending on the values of parameter  $\sigma$ . When  $\sigma = 1$ , then  $X_{inf} = X^*/2$  which is the inflection point of the logistic model.

### 3 Pattern identification in mobile telephony diffusion in EU-15

#### 3.1 Model Identification Results

Mobile Telecommunications has recently developed into a popular innovation of diffusion studies field. In fact, researchers have conducted studies on a national level ([Wright *et al.*, 1997], [Frank, 2003]), a multinational level ([Gruber and Verboven, 2001], [Gruber, 2001]) and on a worldwide level [Dekimpe and Sarvary, 1996]. These, multinational or cross-country studies examine the reasons and dynamics behind the differences in the adoption or diffusion processes of a set of countries. The present approach is trying to identify the existence of standardized patterns in mobile telephony diffusion in EU-15 due to the different time-delay effects between adopters and potential adopters during the contagion process.

The available data express the penetration level of mobile telephony in EU-15 from 1990 until 1992 and has been taken from OECD communication outlook (2000, 2001, and 2002). The proposed model is applied to the available data by using an appropriate non-linear regression algorithm [Skiadas, 1987]. The results for the fifteen countries under investigation are summarized in Table 1.

As it can be seen, the model identification performance is very good since it explains for every country more than 99% of the process variance. The parameter  $\sigma$  is statistically significant for every country showing that the assertion of the existence of time-delay between the awareness and adoption phases is true. Based on the outcomes, the time-delay varies from 0.33 yrs to 1.79 yrs. Figure 1 illustrates the time-delay parameters for each country under investigation. Among the countries with the smaller time-delay parameter are Portugal, France and Greece, while the countries with the bigger time-delay parameter are UK, Luxembourg, Germany, and Denmark, Sweden. It is obvious that a catching-up process is present in the diffusion of mobile telecommunications (Gruber and Verboven, 2001), since the countries with high technology level or countries which belong to the originators of the mobile technology present a bigger time-delay parameter than other countries which develop the industry later on. Finally, three countries present almost symmetric diffusion pattern (inflection point around 50% of the saturation level) while all the others not.

#### 3.2 Time-Delay Effect and Speed of Diffusion

It is interesting to examine the relationship between the time-delay effect and the speed of the diffusion process. A frequently utilized measure for the speed is the reciprocal of characteristic duration, a measure expressing the time required to grow from 10% to 90% of the estimated saturation level. Solving Eq. (10) w.r.t.  $1/t$  yields:

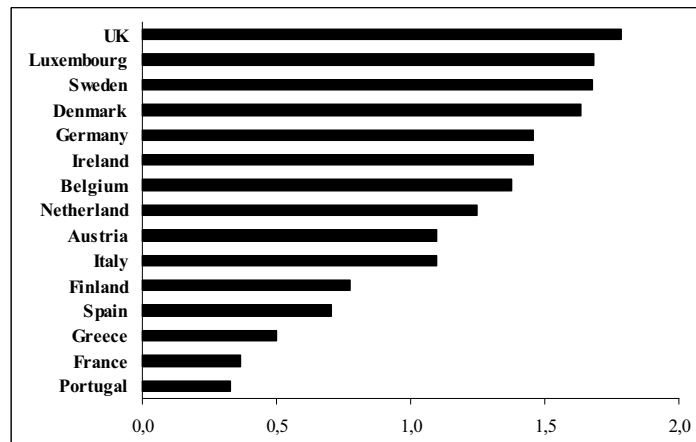
Country	$X_0$	$b^*$	$X^*$	$\sigma$	$V(e_t)$	MSE	Var Expl.	$T$	$X_{inf}$
Austria	0,059 (0,025)	0,701 (0,050)	82,459 (1,026)	0,230 (0,073)	1,648	1,141	99,89%	1,10	56%
Belgium	0,067 (0,007)	0,620 (0,0010)	78,790 (0,354)	0,146 (0,015)	0,110	0,076	99,99%	1,38	57%
Denmark	1,878 (0,451)	0,357 (0,041)	95,520 (15,076)	0,416 (0,340)	2,987	2,068	99,72%	1,64	58%
Finland	1,674 (0,390)	0,447 (0,046)	88,994 (4,198)	0,655 (0,253)	2,267	1,569	99,82%	0,77	49%
France	0,016 (0,06)	0,819 (0,049)	67,193 (1,246)	0,699 (0,135)	0,315	0,218	99,96%	0,37	37%
Germany	0,048 (0,023)	0,652 (0,100)	69,997 (1,404)	0,048 (0,015)	3,938	2,726	99,60%	1,46	57%
Greece	0,229 (0,055)	0,768 (0,044)	89,000 (2,242)	0,617 (0,134)	0,477	0,220	99,97%	0,50	50%
Netherlands	0,046 (0,023)	0,693 (0,056)	74,422 (0,964)	0,137 (0,063)	1,756	1,216	99,85%	1,24	54%
Ireland	0,132 (0,028)	0,589 (0,025)	76,599 (0,595)	0,141 (0,040)	0,531	0,368	99,96%	1,46	56%
Italy	0,221 (0,033)	0,579 (0,019)	94,533 (0,998)	0,364 (0,051)	0,356	0,246	99,98%	1,10	60%
Luxembourg	0,223 (0,085)	0,534 (0,041)	99,556 (2,257)	0,101 (0,049)	4,891	3,386	99,74%	1,68	77%
Portugal	0,030 (0,012)	0,801 (0,053)	85,127 (1,656)	0,738 (0,151)	0,613	0,425	99,95%	0,33	46%
Spain	0,027 (0,010)	0,750 (0,083)	83,563 (2,943)	0,473 (0,200)	2,236	1,548	99,82%	0,70	50%
Sweden	3,150 (0,460)	0,330 (0,029)	99,392 (9,747)	0,447 (0,214)	1,881	1,302	99,84%	1,68	60%
UK	0,204 (0,091)	0,538 (0,101)	81,853 (2,486)	0,039 (0,011)	2,358	8,556	99,04%	1,79	69%

**Table 1.** Parameter Estimates, MSE, and % Variance Explained for the Diffusion of Mobile Telephony in EU-15 (standard errors in parentheses).

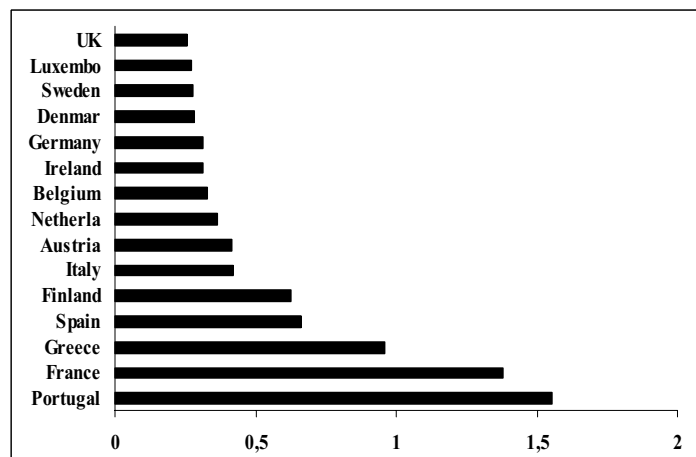
$$SPD = \frac{1}{t} = b^* \cdot \left[ \ln \frac{X_t}{X_0} + \sigma \cdot \ln \frac{X^* - X_t}{X^* - X_0} \right]^{-1} \quad (12)$$

where for each country  $X_0$  represents the 10% of the saturation level and  $X_t$  represents the 90% of the saturation level. Figure 2 illustrates the results concerning the speed of mobile telephony penetration for each country under investigation.

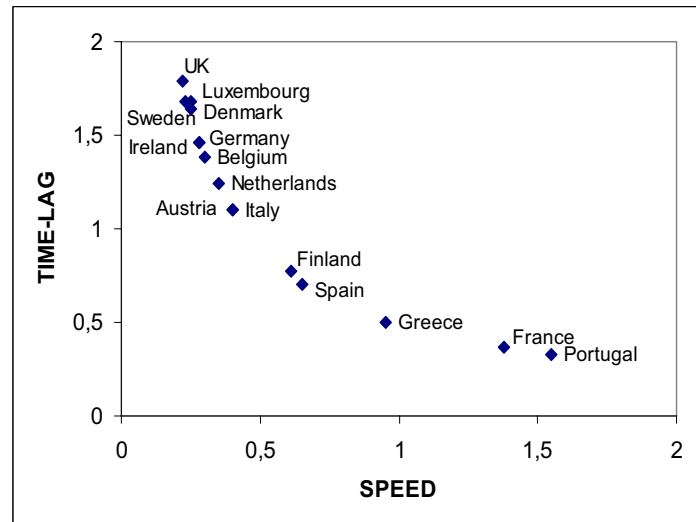
Figure 3 illustrates the mobile telephony speed of EU-15 countries w.r.t. their estimated time-delay effect.



**Fig. 1.** Time-Delay Parameters for EU-15



**Fig. 2.** The Speed of Mobile Telephony Penetration for EU-15



**Fig. 3.** Time-Delay and Speed of Mobile Telephony Diffusion for EU-15

From the above three figures, it can be seen that the time delay effect in the contagion process has a negative impact on product's penetration rate, at least in the medium phases of the diffusion process.

It could be beneficial in future research efforts to identify and include into the model the market factors affecting the magnitude of the time-delay parameter in order that technology marketing functions to be able to facilitate product's marketability.

## 4 Conclusions

This paper proposed a new modeling approach for the investigation of the diffusion of mobile telecommunications services in EU-15. It was found that the proposed model which incorporates the notion of the time-delay between the awareness and the adoption phases of a new product plays an important role in studies of new product penetration in different groups of potential agents. The model was applied to the data of mobile telecommunication in EU-15 and the time-delay effect was used for the ranking of the countries under investigation with respect to their ability to adopt and diffuse the new technology. Furthermore, a new speed index was developed aimed to measure the speed of innovation diffusion. The relationship between the speed of the diffusion and the time-delay effect was studied revealing that they are related in a quadratic mode i.e. as the time-delay effect of diffusion increases the speed of diffusions decreases in a quadratic mode.



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