Time Series Analysis, Control Charts: An Industrial Application

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Abstract. In this work, time series analysis and control charts are used to devise a real-time monitoring strategy in a BTA deep-hole-drilling process. BTA deep-hole-drilling is used to produce holes with high length to diameter ratio, good surface finish and straightness. The process is subject to dynamic disturbances usually classified as either chatter vibration or spiralling. In this work, we will focus on chatter which is dominated by single frequencies. The results showed that the proposed monitoring strategy can detect chatter and that some alarm signals are related to changing physical conditions of the process.
Keywords: Drilling process, Time series, Control charts.

1 Introduction

Deep hole drilling methods are used for producing holes with a high length-to-diameter ratio, good surface finish and straightness. For drilling holes with a diameter of 20 mm and above, the BTA (Boring and Trepanning Association) deep hole machining principle is usually employed. The working principle is shown in Figure 1. The process is subject to dynamic

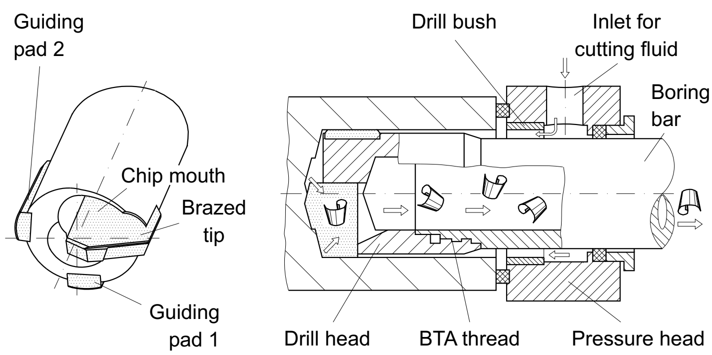

Fig. 1. BTA deep hole drilling, working principle

disturbances usually classified as either chatter vibration or spiralling. Chatter leads to excessive wear of the cutting edges of the tool and may also damage the boring walls. Spiralling damages the workpiece severely. The defect of form and surface quality constitutes a significant impairment of the workpiece. As the deep hole drilling process is often used during the last production phases of expensive workpieces, process reliability is of primary importance and hence disturbances should be avoided. Therefore, it is necessary that a process monitoring system be devised to detect dynamic disturbances.

In this work, we will focus on chatter which is dominated by single frequencies, mostly related to the rotational eigenfrequencies of the boring bar. Therefore, we propose to monitor the amplitude of the relevant frequencies in order to detect chatter vibration as early as possible. Firstly, models that describe the process are reviewed in section 2. In section 3, the proposed monitoring strategy is discussed. Time series analysis is used in section 4 in order to identify the transition to chatter and to check basic assumptions of the application of control charts. Finally, the control charts are applied to real data in section 5.

2 Process models

[Weinert *et al.*, 2002] used the van der Pol equation to describe the transition from stable operation to chatter in one frequency

$$\frac{d^2M(t)}{dt^2} + h(t)(b^2 - M(t)^2)\frac{dM(t)}{dt} + w^2M(t) = W(t), \tag{1}$$

where $t \in [0, \infty)$, $M(t)$ is the drilling torque, $b \in \mathbb{R}$, the frequency $w \in$ [200,2500], $h(t) : \mathbb{R} \to \mathbb{R}$ is an integrable function and $W(t)$ is a white noise process. [Theis, 2004] described the main features of the variation of the amplitudes of the relevant frequencies, using a logistic function. He showed that his approximation is directly connected to the proposed model. In fact, he considered $M(t)$ as a harmonic process

$$M(t) = R(t)cos(w + \phi),$$

where ϕ is the corresponding phase. He showed that

$$2\frac{dR(t)}{dt} + h(t)R(t)\left(b^2 - \frac{R(t)^2}{2}\right) = \frac{W(t)}{w}. \tag{2}$$

is the amplitude-equation for the differential equation in (1) if there is only one frequency present in the process. From equation (2), the observed variation in amplitude of the relevant frequencies may be described by

$$R_t = (1 + a_t)R_{t-1} - a_t b_t R_{t-1}^3 + \varepsilon_t, \tag{3}$$

where a_t and b_t are time varying parameters and ε_t is normally distributed with mean 0 and variance σ_ε^2.

3 Monitoring the residuals

For the monitoring procedure, the model given by equation (3) is approximated by its linear autoregressive part

$$R_t = (1 + a_t)R_{t-1} + \varepsilon_t,$$

and this AR(1) model is used to calculate the residuals. In fact, it is known that the nonlinear term $-a_t b_t R_{t-1}^3$ becomes important when there is chatter. The empirical evidence of this approximation is studied in section 4 using real data. The idea behind residual control charts is if the AR(1) model fits the data well, the residual will be approximately independent. Then, traditional control charts designed to monitor independent data can be applied to the residuals. Generally, residual control charts are designed for processes where stationarity in the steady state is assumed, which means that a unique model parameter for the whole process is used. For this reason a window of the T recent observations is used to estimate parameters a, β and σ_ϵ of the linear regression model

$$R_t = \beta + (1 + a)R_{t-1} + \epsilon_t, \quad (4)$$

where β is included because there is a general shift in the amplitudes after depth 35 mm due to a change in the physical conditions of the process, see section 4.2. The residuals are calculated using

$$e_t = R_t - (1 + \hat{a}_{t-1})R_{t-1} - \hat{\beta}_{t-1}, \quad (5)$$

where $\hat{a}_{t-1}$ and $\hat{\beta}_{t-1}$ are estimates of the regression parameters a and β at time $t-1$. The choice of $\hat{\beta}_{t-1}$ and $\hat{a}_{t-1}$ is motivated by the fact that using the estimated parameters at time t to calculate the residuals and to set the control limits may rather serve to mask changes than to detect them, see [Messaoud *et al.*, 2004b]. In this work, two control charts are considered: the residual Shewhart and a nonparametric EWMA based on standardized sequential ranks.

3.1 The residual Shewhart

The residual Shewhart control chart operates by plotting residuals e_t given by equation (5). It signals that the process is out of control at time t when e_t is outside UCL and LCL, given by

$$LCL = -k\hat{\sigma}_{\varepsilon,t-1} \text{ and } UCL = k\hat{\sigma}_{\varepsilon,t-1},$$

where $\hat{\sigma}_{\varepsilon,t-1}$ is the estimated standard deviation of the regression 4 at time $t-1$ and k is a constant. The choice of k is discussed later. Also, we used $\hat{\sigma}_{\varepsilon,t-1}$ to avoid the masking problem.

For the residual Shewhart charts, it is assumed that the residuals are normally distributed. Thus, the statistical properties of these charts are exact only if this assumption is satisfied. In practice, it is well known that this assumption rarely holds. Therefore, a distribution-free control chart, the EWMA based on sequential ranks, is used to monitor the process.

3.2 The EWMA chart based on sequential ranks

[Hackl and Ledolter, 1992] consider a nonparametric control chart procedure for individual observations that use the "standardized rank" of the observations among the recent group of T observations. For this chart, the sequential rank R_t^* is the rank of e_t among the most recent T ($T > 1$) observations e_t, e_{t-1}, ..., e_{t-T+1}. That is,

$$R_t^* = 1 + \sum_{i=t-T+1}^{t} I(e_t > e_i),$$

where $I(.)$ is the indicator function. The standardized sequential rank $R_t^{(T)}$ is defined as

$$R_t^{(T)} = \frac{2}{T}\left(R_t^* - \frac{T+1}{2}\right).$$

The control statistic Q_t is the exponentially weighted moving averages (EWMA) of standardized ranks, computed as follow

$$Q_t = (1-\lambda)Q_{t-1} + \lambda R_t^{(T)},$$

where $Q_{t,1}$ is a starting value usually set equal to zero, and $0 < \lambda < 1$ is a smoothing parameter. The two sided EWMA chart signals that the process is out-of-control when Q_t is outside $-h$ and h defined to be equal $\pm H\sigma_Q$, where σ_Q and H are the standard deviation of Q_t and a constant, respectively. The choice of h is discussed later. For more details about this chart, see [Hackl and Ledolter, 1992] and [Messaoud *et al.*, 2004b].

4 Time series analysis of the residuals

[Messaoud *et al.*, 2004b] used the two control charts to monitor the variation in amplitudes of frequency 703 Hz, which is among the eigenfrequencies of the boring bar. The data, 1662 observations, are obtained in an experiment with feed $f = 0.185$ mm, cutting speed $v_c = 90$ m/min and amount of oil $\dot{V}_{oil} = 300$L/min. For more details, see [Weinert *et al.*, 2002]. In this experiment chatter is dominated by the frequency 703 Hz. Figure (2) shows the amplitude of frequency 703 Hz. The transition from stable operation to chatter occurs before depth 300 mm. Indeed, by eye inspection, the effect of chatter in this experiment is apparent on the bore hole wall after depth

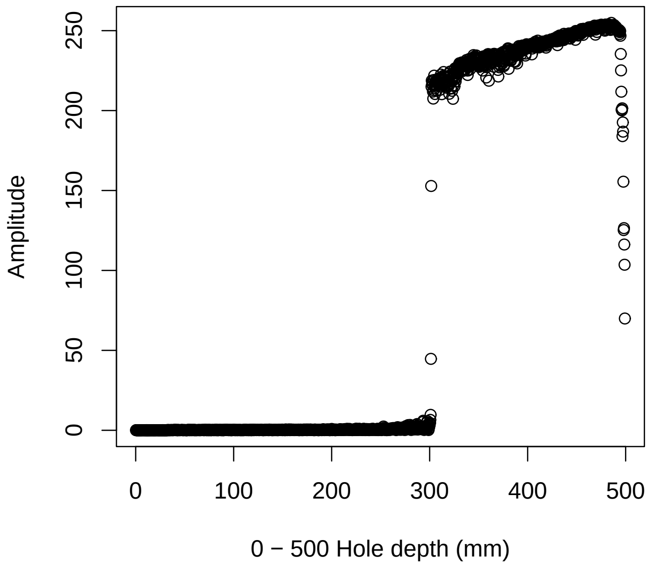

Fig. 2. Amplitude of frequency 703 Hz

300 mm. Therefore, only the first 1000 observations (depth $\leq$ 300 mm) are considered. Figure (3) shows the residuals calculated using equation (5). Note that the first 100 residuals are calculated using

$$e_t = R_t - (1 + \hat{a}_{100})R_{t-1} - \hat{\beta}_{100},$$

where $\hat{a}_{100}$ and $\hat{\beta}_{100}$ are estimates of the regression parameters a and β at time 100.

4.1 Transition from stable state to chatter

In order to investigate the ability of the different control charts to detect chatter, it is important to identify the transition from stable operation to chatter. For this reason, [Messaoud *et al.*, 2004b] studied the mean and variance of frequency 703 Hz. Moreover, the authors applied the Teräsvirta-Lin-Granger statistical test for nonlinear dependence in the residuals, see [Teräsvirta *et al.*, 1993]. As mentioned the nonlinear term $-a_t b_t R_{t-1}^3$ of model given by equation (3) becomes important when the process is unstable. The nonlinearity test is used for residual nonlinear structure, after linear structure has been removed by fitting the AR(1) model. The idea behind this

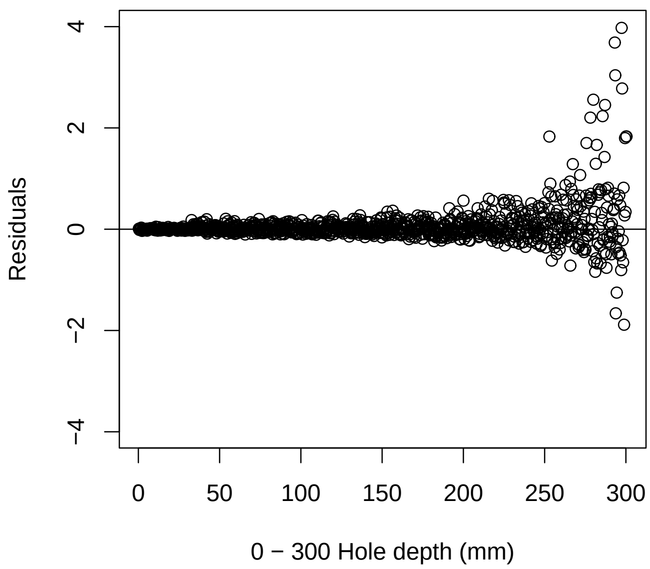

Fig. 3. Plot of the residuals

test is by fitting the linear AR(1) model to the data, the inherent nonlinearity structure has been swept into the residuals. The authors used different time windows of length 100 observations to test for neglected nonlinearity for the regression (3). The results confirms that the nonlinear term $-a_t b_t R_{t-1}^3$ is not important when the process is stable and showed that a change occurs in the process at depth 252.91 mm. This change may indicate the presence of chatter or that chatter will start in a few seconds.

4.2 Independence and normality assumptions of the residuals

[Messaoud *et al.*, 2004b] used the Ljung-Box test in order to check the independence assumption of the residuals. In fact, if the AR (1) model fits the data well, the residuals will be "approximately" independent. This is a basic assumption for the application of the two control charts. In fact, it is known that the performance of control charts is affected by the autocorrelation in the observations. In our process, the presence of autocorrelation in the residuals is destructive to the success of the proposed quality control process. Furthermore, the authors checked the normality assumption using the Shapiro-Wilks test. This assumption is very important only for the Shewhart chart, see section 3. The results shows that the residuals are independent. However, the hypothesis of normality is rejected.

5 Choice of the control charts parameters and results

Knowing that the transition to chatter occurs at depth 252.91 mm, only the first 900 observations (depth $\leq$ 270 mm) are considered for the application of the different control charts. For the reference sample, usually sample of 100-200 observations is used in SPC applications. In this work, the $T = 100$ recent observations $R_{t-T+1}, \ldots, R_t$ are used to estimate the parameters of the AR(1) model and to calculate the residuals. A larger sample cannot be used because the monitoring procedures should start before depth 35 mm (observation 120). In fact, chatter may be observed after that depth because the guiding pads of the BTA tool leave the starting bush, which will be discussed next.

5.1 Choice of the control charts parameters

Usually, the performance of control charts are evaluated by the average run length (ARL). The run length is defined as the number of observations that are needed to exceed the control limit for the first time. The ARL should be large when the process is statistically in-control (in-control ARL) and small when a shift has occurred (out-of-control ARL).

The parameters of the different control charts are selected so that all control charts have the same in-control ARL equal to 370. This choice should avoid many false alarm signals because all control charts are applied to 900 observations. A value $k = 2.95$ is used for the residual Shewhart control charts. For the EWMA chart, we used $\lambda = 0.1$, 0.3 and 0.5. The corresponding values for h are respectively 0.349, 0.629 and 0.786.

5.2 Results

Table 1 shows the out of control signals for depth $\leq$ 270 mm . Table 1 shows that all control charts (except the EWMA charts with λ=0.1 and 0.3) signal at 32 $\leq$ depth $\leq$ 35 mm. As mentioned before the guiding pads leave the starting bush approximately at depth 32 mm, which induce an increase in the process mean and variance for the amplitude of the frequency 703 Hz. This increase explains that all control charts have picked up these changes very quickly. All control charts (except the EWMA charts with λ=0.1 and 0.3) signal at 110 $\leq$ depth $\leq$ 125 mm . It is known that depth 110 mm is approximately the position where the tool enters the bore hole completely. Theis (2004) noted that this might lead to changes in the dynamic process because the boring bar is slightly thinner than the tool and therefore the pressures in the hole may change. The important out of control signals are produced at 250 $\leq$ depth $\leq$ 255 mm. As discussed, it is showed that the transition from stable operation to chatter have occurred at depth 252.91 mm. Therefore, in this experiment chatter may be avoided if corrective actions are taken after this signal.

Table 1. Out of control signals of the different control charts applied to the amplitude of frequency 703 Hz using window length T=100 (depth $\leq$270 mm)

Hole depth (mm)	Observation number	Residual Shewhart	EWMA		
			$\lambda = 0.1$	$\lambda = 0.3$	$\lambda = 0.5$
$\leq$32	$\leq$107	0	0	0	0
32-35	108-117	2	0	0	1
35-45	118-150	7	14	4	2
45-70	151-249	1	0	1	1
70-110	250-366	1	0	0	0
110-125	370-416	1	0	0	1
125-200	417-665	4	8	3	2
200-250	666-832	5	1	0	0
250-255	833-849	2	1	3	2
255-260	850-865	0	0	0	0
260-270	866-898	1	0	0	0
Total		24	24	10	9

Note: The shaded lines refer to the the three physical conditions of the process (i.e., guiding pads leave the starting bush, the tool is completely in the hole and transition from stable operation to chatter)

In this experiment, the EWMA control chart with λ=0.5 is the best, and should be chosen among the three EWMA charts considered in this work. Indeed, only 9 out of control signals are produced and all changes of the physical conditions of the process are detected. In practice, a procedure to choose the smoothing parameter λ is required. As noted in section 5.3, the Residual Shewhart control chart produces more signals than the EWMA control chart with λ=0.5. This may be due to its sensitivity to the normality assumption.

5.3 Multivariate monitoring

In this work, the results showed that chatter can be detected only by monitoring the variation in amplitudes of frequency 703 Hz. This conclusion is expected because this frequency is the relevant frequency in this experiment. However, in practice there are more relevant frequencies and chatter may be observed at the beginning of the drilling process immediately after the guiding pads have left the starting bush, with high and low frequencies, see [Weinert *et al.*, 2002]. Thus, an SPC procedure that monitors all the relevant frequencies is necessary. [Messaoud *et al.*, 2004a] used a multivariate distribution-free EWMA control chart to monitor the drilling process. This chart is based on sequential rank of data depth measures. The results showed that it can detect chatter vibrations.

6 Conclusion

This work showed that using time series analysis and control charts, a reliable on-line monitoring system in the BTA process is proposed. The results showed that the proposed monitoring strategy detect chatter and that some out-of-control signals are related to physical conditions of the process (i. e. guiding pads leave the starting bush, the tool is completely in the hole). Therefore, real-time implementation of this monitoring strategy can be guaranteed.

Acknowledgements

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