A new detection scheme: the sequential Markov detector

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1 Introduction

Automatic detection and tracking (ADT), as performed in radar and sonar, process data $y(t, \omega)$ depending on time variable t and observation variable ω, frequency or direction for instance. ADT decides whether signal from an object to be detected is present or not at any time t and any location ω: this is detection. Once such a positive decision has been taken at some time t_0, ADT has to estimate the location $\omega(t)$ of the signal at further times $t \geq t_0$: this is tracking. The detection step is also named track initiation. Track initiation and tracking are both time association processing. But, because tracking performs only on the data in the vicinity of the tracks, while track initiation has to perform on the whole data domain, tracking may use more computationally intensive algorithms, especially algorithms based on a state model that maps a space of states $\{x\}$ for the detected object to the data domain $\{\omega\}$. Nevertheless this situation may be paradoxical with respect to the fact that deciding that some object is present is at least as important in some applications as once this decision is taken, estimating the state of the object along time. Indeed the detection performance, expressed in terms of detection probability and false alarm probability, is fully achieved by the track initiation step. The continuous increase of the real time computation power let us to envisage the application of the same kind of principle for track initiation like for tracking. In this paper we propose such a new algorithm, a sequential detector based on a hidden Markov model (HMM), that we named the sequential Markov detector (SMD).

The track initiation in most existing radars and sonars rely on a same principle: the P out of N detection. It performs on events detected from single data elements that exceed a detection threshold r_1. The integer N is the duration of the detection test window along the discrete time axis. A signal is detected when there are events at P times at least within the window. This criterion may be refined with the supplementary condition that the mean value of the data corresponding to the highest P events shall be larger than a second detection threshold $r_2 > r_1$. The false alarm probability depends on r_1 and r_2 and on the extent $\delta\omega$ of the test window in the data domain. Increasing $\delta\omega$ results in increasing both the false alarm probability and the capability for accommodating a signal drift along time in the observation domain. In practice, $\delta\omega$ is generally set up at most equal to the resolution of the sensor and the time duration N is set up small enough so that the signal drift cannot exceed this extent. Then N may have to be limited to a few units, a constraint that prevents from taking the full benefit for detection from long signal duration. Whatever be the P out of N variant used in practice, N is most often smaller than 10. This may be also a limit related to the duration of the shortest signal to be detected.

In the "track-before-detect" (TBD) approach for ADT, tentative tracks are formed before being validated. Because the data are integrated on a longer time duration according to some model for the dynamics of the object to be detected, this approach is better suited to low signal-to-noise ratio condition. This processing is most often done on data blocks of fixed duration. In [Tonissen and Evans, 1996], the data are integrated along candidate paths by means of a dynamic programming algorithm. In [Barrett and Holdsworth, 1993], HMM is used for likelihood ratio testing. Sequential detection, which allows for taking a decision about presence or absence of a signal embedded in noise, from a variable number of data frames, is proposed for a constant velocity target model in [Blostein and Richardson, 1994], but the test is truncated and the data still are structured in blocks of fixed duration.

We introduce in this paper a new track initiation scheme combining HMM and sequential detection, named the sequential Markov detector (SMD). HMM allows for testing any path $(x(t))_{t_0 \leq t \leq t_0+\Delta t}$ in the state space from an exact expression of the joint likelihood ratio of this path and the data sequence $(y(t, \omega_x(t)))_{t_0 \leq t \leq t_0+\Delta t}$ along the corresponding path $(\omega(t))_{t_0 \leq t \leq t_0+\Delta t}$ in the data domain. Like the sequential probability ratio test (SPRT) [Marano *et al.*, 2005], SMD does not require fixing Δt. But there is no fixed fail threshold in SMD, which involves a factor exponentially decreasing as a function of Δt controlling an automatic reset process.

We review the principle of HMM detection in part 2 and introduce the sequential Markov detector in part 3. Application of SMD for detection of spectral lines in the time-frequency domain is presented in part 4. For this application, we compare SMD to P out of N by means of a Monte-Carlo simulation in part 5.

2 HMM detection

We assume that the time behaviour of the object to be detected from its signal embedded in background noise is a Markov process taking its values in a finite state space $\{x_1 \dots x_N\}$. Then the a priori probability for any path $X(t_0, \Delta t) = (t_0, \Delta t)_{t=t_0 \dots t_0+\Delta t}$ being the path of the object equals the product of the initial state probability and the probabilities of the transitions between the successive states $x(t-1)$ and $x(t)$ for $t_0+1 \leq t \leq t_0+\Delta t$. The N initial state probabilities $P_n = \mathcal{P}[x(0) = x_n]$ and the N^2 transition probabilities $P_{n,m} = \mathcal{P}[x(t) = x_n | x(t-1) = x_m]$ are known parameters of our Markov model. We assume that the data serie $Y_X(t_0, \Delta t) = (y(t, \omega_x(t)))_{t=t_0 \ldots t_0+\Delta t}$ can be modelled as an independent random process with probability densities p_0 for the background noise and p_1 for the mix of noise and signal:

$$\begin{aligned}
p_0(y(t, \omega_x(t))) &\equiv \mathcal{P}[y(t, \omega_x(t)) | H_0] \\
p_1(y(t, \omega_x(t))) &\equiv \mathcal{P}[y(t, \omega_x(t)) | x(t)]
\end{aligned}$$

where H_0 is the hypothesis that there is no object. Then the joint probability of $X(t_0, \Delta t)$ and $Y_X(t_0, \Delta t)$ can be written as

$$\begin{aligned}
\mathcal{P}[X(t_0, \Delta t), Y_X(t_0, \Delta t)] = {} & \mathcal{P}[y(t_0 + \Delta t, \omega_x(t_0 + \Delta t)) | X(t_0, \Delta t)] \\
& \mathcal{P}[Y_X(t_0, \Delta t - 1) | X(t_0, \Delta t)] \\
& \mathcal{P}[x(t_0 + \Delta t) | x(t_0 + \Delta t - 1)] \\
& \mathcal{P}[X(t_0, \Delta t - 1)]
\end{aligned}$$

The last condition for our model being an HMM [Rabiner and Juang, 1986] is that the information from the state process about the data at some time comes from the state at this time. Then the previous equation takes the following recursive form:

$$\begin{aligned}
\mathcal{P}[X(t_0, \Delta t), Y_X(t_0, \Delta t)] = {} & p_1(y(t_0 + \Delta t, \omega_x(t_0 + \Delta t))) \\
& \mathcal{P}[x(t_0 + \Delta t) | x(t_0 + \Delta t - 1)] \\
& \mathcal{P}[X(t_0, \Delta t - 1), Y_X(t_0, \Delta t - 1)]
\end{aligned}$$

Since we have $\mathcal{P}[Y_X(t_0, \Delta t) | H_0] = \prod_{t=t_0}^{t=t_0+\Delta t} p_0(y(t, \omega_x(t)))$, we get also such a recursive form for the likelihood ratio $\Lambda_{X,Y}(t_0, \Delta t)$ of $(X(t_0, \Delta t), Y_X(t_0, \Delta t))$:

$$\Lambda_{X,Y}(t_0, 0) = \frac{p_1(y(t_0, \omega_x(t_0)))}{p_0(y(t_0, \omega_x(t_0)))} \mathcal{P}[x(t_0)]$$

$$\Lambda_{X,Y}(t_0, \Delta t) = \frac{p_1(y(t_0 + \Delta t, \omega_x(t_0 + \Delta t)))}{p_0(y(t_0 + \Delta t, \omega_x(t_0 + \Delta t)))} \mathcal{P}[x(t_0 + \Delta t) | x(t_0 + \Delta t - 1)] \Lambda_{X,Y}(t_0, \Delta t - 1)$$

For each state x_n, let us consider the maximum value of $\Lambda_{X,Y}(t_0, \Delta t)$ for all paths $X(t_0, \Delta t)$ ending at x_n:

$$\Lambda(x_n, t_0, \Delta t) \equiv \max \{\Lambda_{X,Y}(t_0, \Delta t) | x(t_0 + \Delta t) = x_n\}$$

It can be computed recursively by means of the Viterbi algorithm:

$$\begin{aligned}
\Lambda(x_n, t_0, 0) &= \frac{p_1(y(t_0, \omega_n))}{p_0(y(t_0, \omega_n))} P_n \\
\Lambda(x_n, t_0, \Delta t) &= \frac{p_1(y(t_0 + \Delta t, \omega_n))}{p_0(y(t_0 + \Delta t, \omega_n))} \max_{1 \le m \le N} \{P_{n,m} \cdot \Lambda(x_m, t_0, \Delta t - 1)\}
\end{aligned}$$

ω_n being the location in the data domain corresponding to the state x_n.

By comparing to a threshold the values of $\Lambda(x_n, t_0, \Delta t)$ for $1 \leq n \leq N$, we perform a detection test that, among all tests operating on the same time window, maximises the detection probability for a fixed false alarm probability determined by the detection threshold value. This holds for any signal starting before t_0 and ending after $t_0 + \Delta t$. For some given signal, the best performance is achieved when the processing time window equals the time interval when the signal is present. In practice, this interval is often unknown. A way for handling this problem is to perfom the processing for all possible values of t_0 and Δt. In practice, a trade-off has to be found between the computation cost and the detection performance by taking $(t_0, \Delta t)$ from some reduced subset into the set of the possible values.

3 Sequential Markov detector

SQPRT [Marano *et al.*, 2005] is a detection test that does not require fixing a priori Δt. It computes recursively the likelihood ratio of an i.i.d. data time serie and compares its current value to a downer threshold, the fail threshold, and an upper one, the detection threshold. If the likelihood ratio value is smaller than the fail threshold, H_0 is decided. If it stands between both thresholds, the likelihood ratio is multiplied by the likelihood ratio of the next data element and a new test is performed. If it is higher than the detection threshold, the signal presence hypothesis H_1 is decided. The false alarm probability $P_\mathrm{f} = \mathcal{P}[H_1 decided|H_0]$ relates mainly on the detection threshold value, approximately equal to $P_\mathrm{d}/P_\mathrm{f}$, P_d being the desired detection probability $\mathcal{P}[H_1 decided|H_1]$. The detection probability relates mainly on the fail threshold, approximately equal to $(1 - P_\mathrm{d})/(1 - P_\mathrm{f})$, close to $1 - P_\mathrm{d}$ if $P_\mathrm{f} \ll 1$.

We look now at how SQPRT could be applied to the maximum likelihood ratio $\Lambda(x_n, t_0, \Delta t)$ defined in section 2. Testing $\Lambda(x_n, t_0, \Delta t)$ for detection is equivalent to testing all the likelihood ratio values of the paths $X(t_0, \Delta t)$ ending at state x_n. The number of these paths is growing exponentially as a function of Δt because of the number of state transitions allowed at each time step. So does the false alarm probability of the test performed on the maximum value $\Lambda(x_n, t_0, \Delta t)$. For making this probability independent on Δt, we should decrease by an inverse factor the SQPRT false alarm probability P_f, so increase inversely the detection threshold value $P_\mathrm{d}/P_\mathrm{f}$. Equivalently the threshold value may be kept constant and the likelihood ratio multiplied at each update step by a constant factor K smaller than 1.

In the standard SQPRT, H_0 is definitely decided and the test is ended when the test value goes below the fail threshold approximately equal to $(1 - P_\mathrm{d})/(1 - P_\mathrm{f})$. This is because of the assumption that either H_0 holds for the whole data serie or H_1 does. If the signal may be present only during some time interval within the time interval of the data, the test must be reset once it failed in order to cope with the possibility that the past data might be noise only and that signal might start at some further time. Then a rather logical reset process would be to disregard the past data if, by doing so, the current test value, and consequently the further ones because of the recursive computation, are increased. Such reset process aims to prevent the signal detection from being jeopardized by the noise data before the starting time of the signal.

From the above principles, we can now introduce our new test, the Sequential Markov Detector. Its test value $\Lambda_n(t)$ at each time t and at each point of an HMM state space has the following recursive definition:

$$\Lambda_n(0) = \frac{p_1(y(0,\omega_n))}{p_0(y(0,\omega_n))} P_n$$

$$\Lambda_n(t) = \frac{p_1(y(t,\omega_n))}{p_0(y(t,\omega_n))} \max\{K \max_{1\leq m\leq N}\{P_{n,m}\cdot\Lambda_m(t-1)\}, P_n\}$$

where K is a constant smaller than 1. There is the following relation between $\Lambda_n(t)$ and the maximum likelihood ratio $\Lambda(x_n, t_0, \Delta t)$ presented in section 2:

$$\Lambda_n(t) = K^{t-\tau_n(t)}\Lambda(x_n, \tau_n(t), t-\tau_n(t))$$

where $\tau_n(t)$ is the latest time anterior or equal to t when Λ_n was reset. As discussed above, this relation is the one intended for making the false alarm probability independent on Δt and the $\Lambda_n(t)$ reset condition is that the test value from the current data only $\Lambda(x_n, t, 0)$ is larger than the test value taking the past data into account $K^{t-\tau_n(t-1)}\Lambda(x_n, \tau_n(t-1), t-\tau_n(t-1))$. K is set up so that the following relation holds:

$$\mathcal{P}[K \max_{1\leq m\leq N}\{P_{n,m}\cdot\Lambda_m(t-1)\} > P_n|H_0] = \frac{1}{2}$$

expressing the fact that the probabilities for a path being continued or reset are equal when there is no signal.

$\Lambda_n(t)$ can be computed according to the expression of its above recursive definition. This computation is quite similar to the Viterbi algorithm, except for the reset process. In the next paragraph, we show how to apply it to detection of spectral lines in the time-frequency plane.

4 Application to detection of spectral lines in time-frequency data

The observed data $y(t,f) = |S(t,f)|^2$ are the square magnitude of the output of a time sliding Fourier transform performed on a scalar signal. Hence the observation variable noted previously ω is the frequency, noted from now f.

We define the state as being the pair composed by the frequency of the signal to be detected and its time derivative, which we call the slope:

$$x(t) = (f_x(t), \dot{f}_x(t))$$

We assume that the complex noise component of $S(t, f)$ is zero-mean gaussian with unit variance and that the signal amplitude is constant with signal-to-noise power ratio r_0. Then p_0 and p_1 are homothetic to centered and uncentered χ^2 laws with 2 degrees of freedom:

$$\begin{aligned} p_0(y) &= \exp(-y) \\ p_1(y) &= \exp(-y - r_0) \cdot I_0(2\sqrt{r_0 y}) \end{aligned}$$

with $I_0(z) = \frac{1}{\pi}\int_0^\pi e^{z\cos(\theta)}d\theta$.

Let us be T_{FT} the length of the sliding time window of the Fourier transform and k_t and k_f the coefficients such that the time step of the data $y(t, f)$ equals T_{FT}/k_t and the frequency step of the states equals T_{FT}^{-1}/k_f. Then the slope step is taken equal to $(k_t/k_f)T_{\mathrm{FT}}^{-2}$, the ratio of the frequency step to the time step. So the state space is a finite grid in the real plane with mesh $(T_{\mathrm{FT}}^{-1}/k_f, (k_t/k_f)T_{\mathrm{FT}}^{-2})$. Within this grid, we define the transition probabilities $P_{n,m}$ as following:

$$\begin{aligned} \text{if} \quad & \frac{-1}{2k_f T_{\mathrm{FT}}} \le f_n - f_m - \frac{\dot{f}_n + \dot{f}_m}{2}\frac{T_{\mathrm{FT}}}{k_t} < \frac{1}{2k_f T_{\mathrm{FT}}} \\ \text{then} \quad & P_{n,m} = h\left(\frac{k_f T_{\mathrm{FT}}^2}{k_t}|\dot{f}_n - \dot{f}_m|\right) \\ \text{else} \quad & P_{n,m} = 0 \end{aligned}$$

where h may be any decreasing function such that

$$h(0) + 2\sum_{i=1}^{\infty} h(i) = 1$$

The above relations means that the probability of the transition from the state $(f_m, \dot{f}_m)$ to the state $(f_n, \dot{f}_n)$ is non zero if and only if the frequency change $f_n - f_m$ equals the mean slope value $(\dot{f}_n + \dot{f}_m)/2$ multiplied by the time step T_{FT}/k_t within an error less than half the frequency quantization step T_{FT}^{-1}/k_f. Then the transition probability is a decreasing function of the absolute value of the slope change $\dot{f}_n - \dot{f}_m$. For a given state $(f_m, \dot{f}_m)$ and a given slope change $\dot{f}_n - \dot{f}_m$, there is only one frequency f_n which fulfils the first relation. Then the last relation is equivalent to the condition $\sum_n P_{n,m} = 1$, which expresses the fact that the sum of the probabilities, conditional with respect to some state m, of all its possible successors n, equals 1.

In practice the setting of the function h that determines the transition probabilities may be rather arbitrary because a statistical model for the frequency fluctuation of the signals to be detected is seldom available. The broader is the peak of h at 0, the better is the processing capability to cope with fast fluctuation of the frequency slope, but the lower is the performance achieved on constant frequency slope signals, especially stable frequency signals. The performance decrease on stable frequency signals when the model is changed from a setting suited to them to a setting suited to fluctuating frequency slope signals is illustrated by results shown in the next paragraph.

5 Performance evaluation

We compared the performances of SMD and P out of N detector by means of a Monte-Carlo simulation for detection of spectral lines in magnitude-square FFT data as described in the previous paragraph. The data and the states have the same time and frequency steps with $k_t = 2$ and $k_f = 4$. Note that since k_t is larger than 1, the assumption of time independent data is not valid. This deviation with respect to the HMM theoretical frame, rather usual because the data sampling frequency is above the Shannon bound in many applications, is not expected to have a significant impact on the performance.

We tested two SMD settings having both their signal-to-noise ratio parameter r_0 equal to 0.5 and uniform probability law for the initial states: $P_n = 1/N$ for any n. In the first setting, the slope set is $\{0\}$. Then our model is equivalent to the one where the states are the frequencies and $P_{n,m}$ equals 1 if $n = m$ and 0 otherwise ($h(0) = 1$ and $h(i) = 0$ for $i > 0$). This setting is suited to detection of stable frequency signals. In the second setting, the set of the values for the normalised slope $k_f \dot{f}_n T_{\mathrm{FT}}^2/k_t$ is $\{-2, -1, 0, 1, 2\}$ and the slope change $k_f(\dot{f}_n - \dot{f}_m)T_{\mathrm{FT}}^2/k_t$ takes its value in $\{-1, 0, 1\}$ with an uniform probability law: $h(0) = h(1) = 1/3$ and $h(i) = 0$ for $i > 1$. The SMD output $\Lambda_n(t)$ computed in the state space is projected to the frequency axis according to the relation

$$D(t, f) = \max\{\Lambda_n(t)|f_n = f\}$$

The P out of N detector window covers four frequency channels; so its bandwidth equals the frequency resolution T_{FT}^{-1} of the Fourier transform. The threshold r_1 equals 2. The detector output $D(t, f)$ is computed by placing the $(N, 4)$ window so that one of its points, arbitrarily fixed, is located at the point (t, f) of the time-frequency data. Then $D(t, f)$ equals the mean of the P highest events when the P out of N condition is fulfilled and it equals 0 otherwise. Two P out of N detectors were tested: $(P, N) = (3, 4)$ and $(P, N) = (6, 8)$.

Each value of the detection probability estimate $\hat{P}_{\mathrm{d}}$ in tables 1 to 3 is the mean of the results of 3 independent Monte-Carlo runs, each run involving two files of 2000×1000 time-frequency complex data. One file consists in noise only samples $S_0(t, f)$. The data $S_1(t, f)$ of the second file are samples of the sum of the noise $S_0(t, f)$ and $I = 30$ test signals having the same detection features (signal-to-noise ratio, time duration, frequency fluctuation). Hence 90 test signals were used for each value of the estimate $\hat{P}_{\mathrm{d}}$.

The I test signals in the data $S_1(t,f)$ are located in I non-overlapping time-frequency blocks $[t_{0,\min,i}, t_{0,\max,i}] \times [f_{\min,i}, f_{\max,i}]$ having the same size $\Delta t \times \Delta f$. The computation of $\hat{P}_{\mathrm{d}}$ involves the processing outputs $D_0(t,f)$ and $D_1(t,f)$ from the noise only and signal plus noise input data $S_0(t,f)$ and $S_1(t,f)$. For one Monte-Carlo run, it has the expression:

$$\hat{P}_{\mathrm{d}} = \frac{1}{I^2} \sum_{i=1}^{I} \sum_{j=1}^{I} \left| \begin{array}{l} \max\{D_1(t, f_i(t)) | t_{1,\min,i} \le t \le t_{1,\max,i}\} \qquad > \\ \max\{D_0(t, f_i(t)) | t_{0,\min,j} \le t \le t_{0,\max,j}, f_{\min,j} \le f \le f_{\max,j}\} \end{array} \right|$$

where $|true|$ equals 1, $|false|$ equals 0, $[t_{1,\min,i}, t_{1,\max,i}]$ is the time interval of the ith signal included in $[t_{0,\min,i}, t_{0,\max,i}]$, and $f_i(t)$ is the signal frequency determined by the relation

$$f_i(t) = \arg\max_{\mathrm{f}} \left\{ |S_1(t,f) - S_0(t,f)| \Big| f_{\min,i} \le f \le f_{\max,i} \right\}$$

The I detection threshold values are the maximum output values on the I noise-only data blocks. Hence they relate to a mean number of false alarms per data block equal to 1. We define the false alarm probability $P_{\mathrm{f,in}}$ as the ratio of the output false alarm rate to the input rate of independent data. Each data block containing $\Delta t \times \Delta f$ independent data elements, we have:

$$P_{\mathrm{f,in}} = \frac{1}{\Delta t \times \Delta f}$$

Each data block consists in 300 time lines of 200 adjacent frequency channels. So we have $P_{\mathrm{f,in}} = 1/(300 \times T_{\mathrm{FT}}/k_t)/(200 \times T_{\mathrm{FT}}^{-1}/k_f) = 1.3 \times 10^{-4}$.

Defining the false alarm probability with respect to the input data allows a fair comparison between detectors having different rates of independent decisions. In order to validate the above method, the detection probability of the integrator of time constant equal to the signal duration T_{sig} was estimated with the above method in the cases of stable frequency signals with $T_{\mathrm{sig}} = 100 \times T_{\mathrm{FT}}$ (Table 1) and $T_{\mathrm{sig}} = 10 \times T_{\mathrm{FT}}$ (Table 2). The well-known ROC curves for the Rice case give the theoretical value of the signal-to-noise ratio r corresponding to the measured detection performance $(\hat{P}_{\mathrm{d}}, P_{\mathrm{f,out}})$, $P_{\mathrm{f,out}} \approx (T_{\mathrm{sig}}/T_{\mathrm{FT}}) \times P_{\mathrm{f,in}}$ being the false alarm probability at the detector output as considered in these curves. This theoretical value is given in parentheses below the $\hat{P}_{\mathrm{d}}$ value in tables 1 and 2. The difference between both signal-to-noise ratio values was always found smaller than 1 dB. This difference may be caused not only by the estimation error on $\hat{P}_{\mathrm{d}}$ but also likely by the fact that in our test we have a random detection threshold and a fixed false alarm rate, while the ROC curves hold for a deterministic threshold and a random false alarm rate.

From the results in tables 1 and 2, the cost of not knowing the signal duration in stable state SMD with respect to the performance achieved by

$10\log_{10}(r)$	-5	-4	-3	-2	-1	0	1	2	3
integrator $100 \times T_{\mathrm{FT}}$ (SNR(dB) for $P_{\mathrm{f,out}} = 0.013$)	0.53 (-5.9)	0.83 (-4.5)	0.93 (-3.6)	0.98 (-2.8)					
SMD – stable states	0.30	0.55	0.74	0.90	0.96				
SMD – 5 slopes $h(0) = h(1) = 1/3$		0.30	0.46	0.60	0.79	0.97			
3 out of 4					0.31	0.40	0.50	0.67	0.90

Table 1. $\hat{P}_{\mathrm{d}}$ for $P_{\mathrm{f,in}} = 1.3 \times 10^{-4}$ – Stable frequency – $T_{\mathrm{sig}} = 100 \times T_{\mathrm{FT}}$.

$10\log_{10}(r)$	2	3	4	5	6	7
integrator $100 \times T_{\mathrm{FT}}$ (SNR(dB) for $P_{\mathrm{f,out}} = 0.013$)	0.54 (1.2)	0.88 (3.0)	0.92 (3.2)			
SMD – stable states	0.47	0.84	0.90	0.99		
SMD – 5 slopes $h(0) = h(1) = 1/3$		0.49	0.63	0.89	0.95	0.99
3 out of 4			0.33	0.66	0.79	0.95

Table 2. $\hat{P}_{\mathrm{d}}$ for $P_{\mathrm{f,in}} = 1.3 \times 10^{-4}$ – Stable frequency – $T_{\mathrm{sig}} = 10 \times T_{\mathrm{FT}}$.

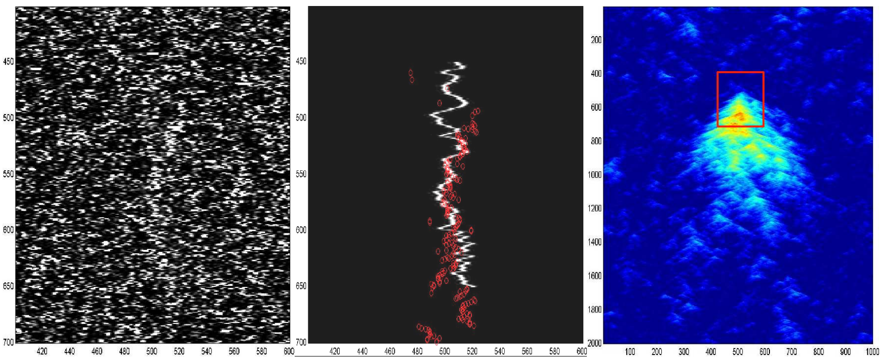
the time integrator with time constant equal to the signal duration appears being close to 1 dB when the signal contains 100 independent samples and smaller than 1 dB when it contains 10 independent samples. The ability to perform also on fluctuating frequency slope signal with the second SMD setting is provided with an additional cost standing between 1 dB and 2 dB in detection of stable frequency signals. Then the gain with respect to 3 out of 4 detection stands between 3 and 4 dB for 100 independent sample signal and between 1 and 2 dB for 10 independent sample signal.

Performances on fluctuating frequency signals are presented in Table 3. The frequency fluctuation is gaussian with standard deviation σ_f taking values 0, T_{FT}^{-1}, $2T_{\mathrm{FT}}^{-1}$ and $3T_{\mathrm{FT}}^{-1}$. The time length of the fluctuation correlation equals $15 \times T_{\mathrm{FT}}$. As expected, the performance from the integrator with time constant equal to signal duration is much sensitive to signal frequency fluctuation. 6 out of 8 detector performs better than 3 out of 4 detector only when σ_f is smaller than $2T_{\mathrm{FT}}^{-1}$. This illustrates the fact that the N parameter of the P out of N detector is limited by the expected drift of the signal to be detected. Anyway SMD still performs significantly better than P out of N in all test cases.

An example of input and output data is displayed on Figure 1 where, for illustration clarity, only one signal is embedded in noise, the data format being the same than the one described above. The signal features are $10\log_{10}(r) = -1$ dB, $T_{\mathrm{sig}} = 100 \times T_{\mathrm{FT}}$ and $\sigma_f = 3T_{\mathrm{FT}}^{-1}$. In practice, the SMD output would be reset in the vicinity of a detection and a specific tracking process should be started for maintenance and termination testing of the newly validated track. This further tracking process, not performed in this work

$\sigma_f \times T_{\mathrm{FT}}$	0	1	2	3
integrator $100 \times T_{\mathrm{FT}}$	1.00	0.58	0.21	0.14
SMD – 5 slopes $h(0) = h(1) = 1/3$	0.79	0.75	0.61	0.53
3 out of 4	0.31	0.28	0.23	0.17
6 out of 8	0.48	0.34	0.20	0.10

Table 3. $\hat{P}_{\mathrm{d}}$ for $P_{\mathrm{f,in}} = 1.3 \times 10^{-4}$ – Fluctuating frequency – $T_{\mathrm{sig}} = 100 \times T_{\mathrm{FT}}$ – $10 \log_{10}(r) = -1\mathrm{dB}$.



Signal+ noise input $|S_1(t,f)|^2$ Zoom 300 times $\times$ 200 frequencies

Track: amplified signal $|S_1(t,f) - S_0(t,f)|^2$ Red circles: frequency bin in $[1, 1000]$ of maximum SMD output $D_1(t,f)$ Zoom 300 times $\times$ 200 frequencies

SMD signal + noise output $D_1(t,f)$ Full time-frequency range 2000 times $\times$ 1000 frequencies Zoom window in red

Fig. 1. Example of input and output data.

entirely devoted to the track initiation problem, would avoid the spreading of the SMD output peaks seen on Figure 1.

6 Conclusion

We presented a new track initiation method named the Sequential Markov Detector. It is a “track-before-detect” processing which combines HMM tracking and sequential detection. The detection test value is the a posteriori likelihood ratio weighted by a factor exponentially decreasing as a function of the time duration of the tested path in the state space. It is reset when taking into account only the current data provides a larger value.

This new detector was shown to perform significantly better than the usual P out of N detector for spectral line detection from time-frequency data. The margin for further performance improvement from the same kind of data and the same a priori information about the signal is likely small since the detection loss on a stable spectral line with respect to the constant frequency integrator matched to the signal duration is at most 3 dB for signal bandwidth-time product at most equal to 100, while at least a part of this loss is the unavoidable cost for SMD ability to perform on unknown-duration unstable-frequency signal. Further research should rather to look at how exploiting richer data, for instance complex spectral data instead of magnitude data, or more accurate a priori information within the state model.

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